Competitive Markets

(Mas-Colell Chpt 10)

Notation

Goods - L total; individual good represented by $\ell = 1, 2, ..., L$

Initial Endowment - $\omega_\ell \geq 0$; amount of good ℓ initially available in the economy

Consumers - I total; individual consumer represented by $i = 1, 2, ..., I$

Consumption Set - $X_i \subset R^L$

Consumption Bundle - $\mathbf{x}_i = (x_{1i}, x_{2i}, ..., x_{L\,i}) \in X_i$

Utility Representation - $u_i(\mathbf{x}_i)$; also called utility function

Firms - J total; individual firms represented by $j = 1, 2, ..., J$

Production Set - $Y_j \subset R^L$

Production Bundle - $\mathbf{y}_j = (y_{1j}, y_{2j}, ..., y_{L\,j}) \in Y_j$, where $y_{\ell\,j} < 0$ if good ℓ is an input and > 0 if good ℓ is an output

Definitions

Feasible Allocation - specification of consumption vector for each consumer and production vector for each firm $(\mathbf{x}_1, ..., \mathbf{x}_I, \mathbf{y}_1, ..., \mathbf{y}_J)$ such that $\mathbf{x}_i \in X_i$, $\mathbf{y}_j \in Y_j$ and the total consumption of each good ℓ doesn't exceed (i.e., ≤) the initial endowment plus the amount produced:

$$\sum_{i=1}^{I} x_{\ell i} \leq \omega_\ell + \sum_{j=1}^{J} y_{\ell j} \quad \forall \quad \ell = 1, 2, ..., L$$

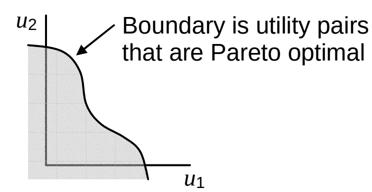
Pareto Optimal (PO) Allocation - feasible allocation $(\mathbf{x}_1, ..., \mathbf{x}_I, \mathbf{y}_1, ..., \mathbf{y}_J)$ is Pareto optimal (or **Pareto efficient, PE**) if there is no other feasible allocation $(\mathbf{x}_1', ..., \mathbf{x}_I', \mathbf{y}_1', ..., \mathbf{y}_J')$ such that

a. $u_i(\mathbf{x}_i') \geq u_i(\mathbf{x}_i)$ $\forall$ $i = 1,..., I$ (i.e., every consumer gets at least as much), and

b. $u_i(\mathbf{x}_i') > u_i(\mathbf{x}_i)$ for some i (i.e., some consumers do better)

That is, you can't make anybody better off without making someone else worse off

PO Allocations form the northeast boundary of the utility possibility set



Utility Possibility Set - set of all combinations of attainable levels of utility for all consumers:

$U = \{(u_1, ..., u_I) \in R^I : \exists$ a feasible allocation $(\mathbf{x}_1, ..., \mathbf{x}_I, \mathbf{y}_1, ..., \mathbf{y}_J)$ s.t. $u_i \leq u_i(\mathbf{x}_i)$ $\forall$ $I = 1, ..., I)\}$

Not Equity - PO doesn't ensure allocation is in any sense equitable (fair)

Minimal Test - PO serves as important minimal test for desirability of an allocation; says that there is no waste in the allocation of society's resources (although not everyone may be happy with that allocation)

Competitive Market - 3 assumptions

1. Initial Endowment of each good ℓ is fully distributed across consumers

   $\omega_{\ell i}$ = consumer i's endowment of good ℓ ∴total endowment $\omega_\ell = \sum_{i=1}^{I} \omega_{\ell i}$

   $\boldsymbol{\omega}_i = (\omega_{1i}, \omega_{2i}, ..., \omega_{Li})$; vector of consumer i's endowment of each good

2. Consumers Own Firms so they take a share of the profits

   θ_{ij} = share (%) of firm j's profit owned by consumer i; $\sum_{i=1}^{I}\theta_{ij} = 1 \; \forall \; j = 1, ..., J$

3. Markets Exist and producers and consumers are price takers for each good ℓ
   a. Each agent makes decisions assuming price doesn't change based on the decision (small relative to size of market so they're price takers)
   b. No public goods (because a market exists)

Competitive Equilibrium (CE) - also called **Walrasian** equilibrium; feasible allocation ($\mathbf{x}_1$*, ..., $\mathbf{x}_I$*, $\mathbf{y}_1$*, ..., $\mathbf{y}_J$*) and price vector $\mathbf{p}^* = (p_1^*, ..., p_L^*) \in R^L$ that satisfy the following conditions:

1. Profit Maximization - $\mathbf{y}_j^*$ solves Max $\mathbf{p}^*\cdot\mathbf{y}_j \; \forall \; j = 1, ..., J$

   Remember that $y_{\ell j} < 0$ if good ℓ is an input so $\mathbf{p}^*\cdot\mathbf{y}_j$ = revenue - cost = profit

2. Utility Maximization - $\mathbf{x}_i^*$ solves Max $u_i(\mathbf{x}_i)$ s.t. $\mathbf{p}^*\cdot\mathbf{x}_i \le \mathbf{p}^*\cdot\boldsymbol{\omega}_i + \sum_{j=1}^{J}\theta_{ij}\,(\mathbf{p}^*\cdot\mathbf{y}_j) \; \forall \; i = 1, ..., I$

   Value of consumption must not exceed each consumer's endowment (wealth) plus his share of profit from each firm (income)

3. Market Clearing - $\sum_{i=1}^{I} x_{\ell i}^* = \omega_\ell + \sum_{j=1}^{J} y_{\ell j}^* \; \forall \; \ell = 1, ..., L$

   First two conditions just say the agents (producers and consumers) seek to do as well as they can for themselves; this one says that the desired consumption and production levels identified in the first two conditions must be mutually compatible (i.e., supply = demand)

Results - two results follow directly from CE

A. Normalize Prices - if allocation ($\mathbf{x}_1$*, ..., $\mathbf{x}_I$*, $\mathbf{y}_1$*, ..., $\mathbf{y}_J$*) and price vector $\mathbf{p}^* \gg \mathbf{0}$ constitute a CE, then so do allocation ($\mathbf{x}_1$*, ..., $\mathbf{x}_I$*, $\mathbf{y}_1$*, ..., $\mathbf{y}_J$*) and price vector $\lambda\mathbf{p}^*$ for any scalar $\lambda > 0$

   Relative Prices - if all prices multiplied by positive constant, there's no difference in amount produced or consumed; only relative prices matter

   Numeraire - without loss of generality, we can always normalize by setting one good's price equal to 1 (the *numeraire*)

B. Last Good Market Clearing - if allocation ($\mathbf{x}_1, ..., \mathbf{x}_I, \mathbf{y}_1, ..., \mathbf{y}_J$) and price vector $\mathbf{p} \gg \mathbf{0}$ satisfy the market clearing condition $\forall$ goods $\ell \ne k$, and if every consumer's budget constraint holds with equality, then market clearing condition also holds for good k

   Proof:

   Add budget constraints: $\sum_{i=1}^{I}\mathbf{p}\cdot\mathbf{x}_i = \sum_{i=1}^{I}\mathbf{p}\cdot\boldsymbol{\omega}_i + \sum_{i=1}^{I}\sum_{j=1}^{J}\theta_{ij}\,(\mathbf{p}\cdot\mathbf{y}_j)$

   Note that share of profit drops out because $\sum_{i=1}^{I}\theta_{ij} = 1 \; \therefore$

   $$\sum_{i=1}^{I}\mathbf{p}\cdot\mathbf{x}_i = \sum_{i=1}^{I}\mathbf{p}\cdot\boldsymbol{\omega}_i + \sum_{j=1}^{J}\mathbf{p}\cdot\mathbf{y}_j$$

   Break it out by good: $\sum_{\ell=1}^{L}\sum_{i=1}^{I}p_\ell x_{\ell i} = \sum_{\ell=1}^{L}\sum_{i=1}^{I}p_\ell\boldsymbol{\omega}_\ell + \sum_{\ell=1}^{L}\sum_{j=1}^{J}p_\ell y_{\ell j}$

Rearrange terms to get all $\ell \neq k$ on left hand side:

$$\sum_{\ell \neq k} p_\ell \left(\sum_{i=1}^{I} x_{\ell i} - \omega_\ell - \sum_{j=1}^{J} y_{\ell j} \right) = -p_k \left(\sum_{i=1}^{I} x_{ki} - \omega_k - \sum_{j=1}^{J} y_{kj} \right)$$

Note that left hand side equals zero because of the market clearing condition:

$$0 = -p_k \left(\sum_{i=1}^{I} x_{ki} - \omega_k - \sum_{j=1}^{J} y_{kj} \right)$$

$\therefore$ since $p_k > 0$, we have $\sum_{i=1}^{I} x_{ki} = \omega_k + \sum_{j=1}^{J} y_{kj}$ (market clearing for good k)

Partial Equilibrium Model

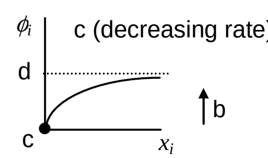
Intuition - looking at market for goods that constitute small part of overall economy $\therefore$ consumer doesn't spend a lot in that market which allows 2 important simplifications:

1. No Income Effect - only small fraction of wealth needed to offset increased price (or added by decreased price) so there's little spill over into other markets
2. No Substitution Effect - prices of other goods should be approximately unaffected by changes in this market

Result - we can look only at the market of interest and ignore the effects on the other markets

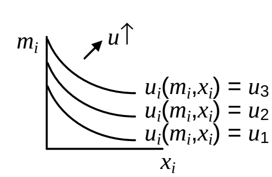
Two Good Case - results in partial equilibrium being equivalent to general equilibrium

1. Two Goods - good under study (good ℓ with price p) and numeraire good (with price = 1)
   $x_i \geq 0$ is consumer i's consumption of good ℓ
   m_i is consumer i's consumption of numeraire good (not restricted for simplicity; avoids boundary solutions)
   Note that we can have the numeraire representing the composite of all other goods (with m standing for the total money expenditure on these other goods)
2. Quasilinear Utility - each consumer has utility function of form: $u_i(m_i, x_i) = m_i + \phi_i(x_i)$
   a. ϕ_i is twice differentiable
   b. $\phi_i' > 0$ (always increasing)
   c. $\phi_i'' < 0$ (increasing at decreasing rate)
   d. ϕ_i bounded from above
   e. Normalize: $\phi_i(0) = 0$



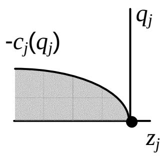
   Convex Indifference Curves - Proof: level curves of $u_i(m_i, x_i)$ in (x_i, m_i) plane are of form: $m_i + \phi_i(x_i) = u$; to be convex, need utility representation $u_i(m_i, x_i)$ to be quasiconcave (i.e., bordered Hessian of $u_i(m_i, x_i)$ to be negative definite):
   $u_1 = 1$ $\quad u_{11} = 0$ $\quad u_2 = \phi_i'$ $\quad u_{22} = \phi_i''$ $\quad u_{12} = u_{21} = 0$

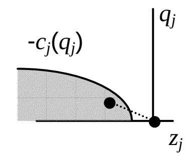
   Need $\mathbf{BH} = \begin{vmatrix} 0 & u_1 & u_2 \\ u_1 & u_{11} & u_{12} \\ u_2 & u_{21} & u_{22} \end{vmatrix} = u_1 u_2 u_{12} - u_2^2 u_{11} - u_1^2 u_{22} > 0$



   $\mathbf{BH} = 1(\phi_i')(0) - (\phi_i')^2(0) - 1^2(\phi_i'') = -\phi_i'' > 0 \Rightarrow \phi_i'' < 0$ (which is correct)
3. Production - firm j produces $q_j \geq 0$ of good ℓ using at least $c_j(q_j)$ (that's the cost function) of numeraire good m; let the amount the firm uses be z_j
   Production Set - $Y_j = \{(-z_j, q_j): q_j \geq 0 \text{ and } z_j \geq c_j(q_j)\}$
   a. c_j is twice differentiable
   b. $c_j' > 0$ (positive marginal cost)

c. $c_j'' \geq 0$ (non-decreasing marginal cost)





d. If $c_j(0) = 0$, production set is convex

d. If $c_j(0) > 0$ (i.e., **fixed cost**), production set is not convex if firm does not incur fixed cost if it chooses not to produce (i.e., $(0,0) \in Y$)

4. Initial Endowments -

Good ℓ - none so it must all be produced; this is just for simplicity so we focus on production and consumption (not inventory)

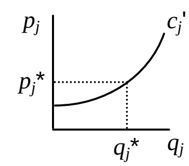
Good m - endowment only to consumers (just as we assumed before on bottom of page 1)

ω_{mi} is amount of good m endowed to consumer i

$\omega_m = \sum_{i=1}^{I} \omega_{mi}$ is total endowment of good m to society

Short-Run Analysis - J (# firms) is fixed; if $c_j(0) > 0$ (i.e., fixed cost), these costs are sunk

CE Step 1 - Profit Maximization - Max $p^*q_j - c_j(q_j)$ s.t. $q_j \geq 0$



Kuhn-Tucker Condition - writing out all steps... utility max will skip this step

$q_j > 0 \Rightarrow d\pi/dq_j = p^* - c_j'(q_j) = 0$

$q_j = 0 \Rightarrow d\pi/dq_j = p^* - c_j'(q_j) \leq 0$

We can rewrite these as a single statement: $p^* \leq c_j'(q_j^*)$, with equality if $q_j^* > 0$

Note: profit comes from $\mathbf{p} \cdot \mathbf{y}_j = (p^*,1) \cdot (q_j,-z_j) = p^*q_j - z_j = p^*q_j - c_j(q_j)$

CE Step 2 - Utility Maximization - Max $m_i + \phi_i(x_i)$ s.t. $m_i + p^*x_i = \omega_{mi} + \sum_{j=1}^{J} \theta_{ij} \pi_j^*$ & $x_i \geq 0$

Note: $\pi_j^* = p^*q_j^* - c_j(q_j^*)$; not writing it out because it's irrelevant to consumer's decision

Constraint is actually $\leq$, but since we have a well behaved problem, we know the solution will be on the boundary of the feasible set of alternatives (i.e., =)

Substitute out m_i from the constraint so problem becomes:

Max $\phi_i(x_i) - p^*x_i + \omega_{mi} + \sum_{j=1}^{J} \theta_{ij} \pi_j^*$ s.t. $x_i \geq 0$

Kuhn-Tucker Condition - $\phi_i'(x_i^*) \leq p^*$, with equality if $x_i^* > 0$

Notes:

Solution is completely independent of $\omega_{mi} + \sum_{j=1}^{J} \theta_{ij} \pi_j^*$ (wealth)

There is an implied m_i^* (have to go back and solve for it)

CE Step 3 - Market Clearing - $\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$

Summary -

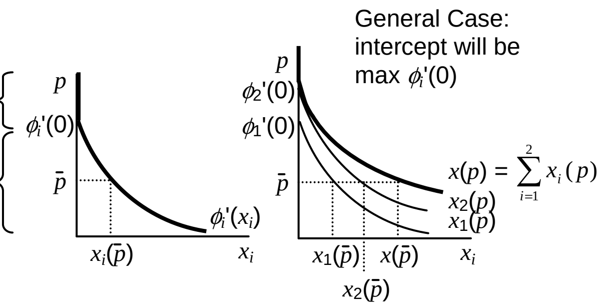
Profit Max... $p^* \leq c_j'(q_j^*)$, with equality if $q_j^* > 0$

Utility Max... $\phi_i'(x_i^*) \leq p^*$, with equality if $x_i^* > 0$

Market Clearing... $\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$

Note: equilibrium allocation (x_i^*) and price (p^*) are independent of initial distribution of endowments (ω_{mi}) and ownership shares (θ_{ij})

Aggregate Demand - also called market demand; can invert the utility maximization solution to get x_i as a function of price



$$x_i(p) = \begin{cases} 0 & p \geq \phi_i'(0) \\ x_i & \text{solving } p = \phi_i'(x_i) \text{ if } p \leq \phi_i'(0) \end{cases}$$

From $p = \phi_i'(x_i)$ we differentiate:

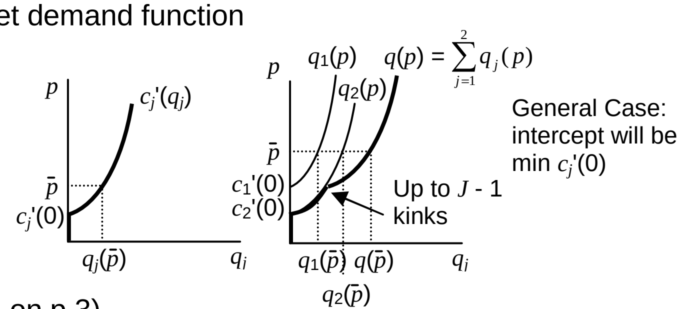
$dp = \phi_i''(x_i)dx_i \;\Rightarrow\; dx_i/dp = 1/\phi_i''$

$\therefore$ slope of $x_i(p) = 1/\phi_i'' < 0$ (see assumption 2 on p.3)

Horizontal Sum - get market demand by summing individual demands: $x(p) = \sum_{i=1}^{I} x_i(p)$

result is <u>downward sloping</u>, <u>continuous</u> market demand function

Aggregate Supply - also called market supply; can invert the profit maximization solution to get q_j as a function of price



$$q_j(p) = \begin{cases} 0 & p \leq c_j'(0) \\ q_j & \text{solving } p = c_j'(q_j) \text{ if } p \geq c_j'(0) \end{cases}$$

From $p = c_j'(q_j)$ we differentiate:

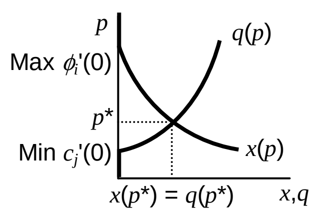
$dp = c_j''(q_j)dq_j \;\Rightarrow\; dq_j/dp = 1/c_j''$

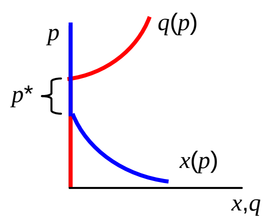
$\therefore$ slope of $q_j(p) = 1/c_j'' \geq 0$ (see assumption 3 on p.3)

Horizontal Sum - get market supply by summing individual supplies: $q(p) = \sum_{j=1}^{J} q_j(p)$

result is <u>upward sloping</u>, <u>continuous</u> market supply function (**Note:** could have up to J - 1 points of non-differentiability ["kinks"])

Non-Trivial Case - Max $\phi_i(0) \geq$ Min $c_j'(0)$... assumption that ϕ is bounded above guarantees $x(p)$ goes to zero so AS and AD cross $\therefore$ market clearing condition for CE exists and is unique





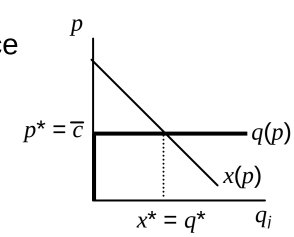
Trivial Case - Max $\phi_i(0) <$ Min $c_j'(0)$... AD and AS do not intersect with x or $q > 0$ (i.e., no product ℓ produced); there are multiple equilibria with $x = q = 0$ and a range of p^* in between Max $\phi_i(0)$ and Min $c_j'(0)$

Constant Marginal Cost - look at "weakly" part of weakly convex cost curve (i.e., $c_j'' = 0$... linear marginal cost curve: $c_j = \bar{c}q_j$ $\forall$ firms $j = 1, ..., J$... assuming each firm has same marginal cost in this case)

Different Costs? - if firms had different, constant marginal costs, firm with lowest cost would monopolize the market because it can produce an infinite amount at a lower cost then all the other firms

Supply Correspondence - now supply isn't a function, but a correspondence

$$q_j = \begin{cases} 0 & p < \bar{c} \\ [0,\infty] & p = \bar{c} \\ \infty & p > \bar{c} \end{cases}$$



Non-Trivial Case - have unique p* and x_j*, but not unique q_j*, only unique $\sum_{j=1}^{J} q_j$ *; all firms make zero profit so it doesn't matter how we split output ("trivial" multiplicity); usually look at symmetric equilibrium (all firms produce same amount)

Interpreting AD & AS - invert aggregate demand and supply curves back to interpret them

Marginal Social Benefit - invert $x(p)$ gives marginal social benefit (value) of corresponding level of output; solve for p in terms of x: $p(x)$

$$p(x) = \begin{cases} \phi_i'(x_i) & \text{for any consumer making purchases (i.e., } x_i > 0\text{);} \\ 0 & \text{for those with } x_i = 0\text{; we know } \phi i'(0) < p \end{cases}$$

In English - for all those consumers purchasing good ℓ (i.e., $x_i > 0$), they have marginal benefit ($\phi_i'(x_i)$) equal to the market price (follows from utility maximization); $\therefore$ all consumers who buy ℓ value it at the same level (p); those who don't purchase it have marginal benefit less than price

Purpose of Demands - $x(p)$ is <u>positive</u> (behavioral): given p, this is how much people will buy; $p(x)$ is <u>normative</u>: makes statements about desirability of CE

Marginal Social Cost - do the same for supply (invert $q(p)$ to get $p(q)$... Mas-Colell writes it as $c'(q)$; gives marginal social cost of corresponding level of output

$$p(q) = \begin{cases} c_j'(q_j) & \text{for any firm producing good } \ell \text{ (i.e., } q_j > 0\text{);} \\ 0 & \text{for those with } q_j = 0\text{; we know } cj'(0) > p \end{cases}$$

In English - for all those firms purchasing good ℓ (i.e., $c_j > 0$), they have marginal cost ($c_j'(q_j)$) equal to the market price (follows from profit maximization); $\therefore$ all firms that produce ℓ have same marginal cost (p); those who don't produce it have marginal cost greater than price

Equilibrium - MSB = MSC

Normative Analysis of Short-Run

Important Result - quasilinear preferences imply boundary of utility possibilities set is linear with slope -1 in every direction; this means utility is transferable because consumption allocations differ only in the distribution of the numeraire good (m) among consumers

<u>Proof</u>:

Consider any arbitrary <u>given</u> (fixed) production and consumption levels of good ℓ:

$$(\bar{\mathbf{x}}, \bar{\mathbf{q}}) = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_I, \bar{q}_1, \bar{q}_2, \ldots, \bar{q}_J)$$

Amount of numeraire (i.e., good m) available for distribution to consumers is amount initially available (ω_m) minus amount used in production:

$$\boldsymbol{\omega}_m - \sum_{j=1}^{J} c_j(\bar{q}_j)$$

Let τ_i be the proportion of numeraire available for distribution that is given to consumer i; note: $\sum_{i=1}^{I} \tau_i = 1$

The utility enjoyed by consumer i is: $\phi_i(\bar{x}_i) + \tau_i \left[\boldsymbol{\omega}_m - \sum_{j=1}^{J} c_j(\bar{q}_j) \right]$

Conditional Utility Possibilities Set - each consumer can have at least as much utility shown above given the fixed production and consumption levels $(\bar{\mathbf{x}}, \bar{\mathbf{q}})$; that set of all these utilities is:

$$\left\{(u_1,u_2,\ldots,u_I): u_i \le \phi_i(\bar{x}_i) + \tau_i\left[\boldsymbol{\omega}_m - \sum_{j=1}^{J} c_j(\bar{q}_j)\right] \ \forall\ i\right\}$$

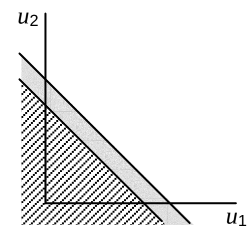
This can be rewritten by adding the utilities:

$$\left\{(u_1,u_2,\ldots,u_I): \sum_{i=1}^{I} u_i \le \sum_{i=1}^{I}\phi_i(\bar{x}_i) + \sum_{i=1}^{I}\tau_i\left[\boldsymbol{\omega}_m - \sum_{j=1}^{J} c_j(\bar{q}_j)\right]\right\}$$

Note that $\boldsymbol{\omega}_m - \sum_{j=1}^{J} c_j(\bar{q}_j)$ has no I terms so it can be moved outside the summation, leaving $\sum_{i=1}^{I}\tau_i = 1$, $\therefore$

$$\left\{(u_1,u_2,\ldots,u_I): \sum_{i=1}^{I} u_i \le \sum_{i=1}^{I}\phi_i(\bar{x}_i) - \sum_{j=1}^{J} c_j(\bar{q}_j) + \boldsymbol{\omega}_m\right\}$$

The frontier of the conditional utility possibilities set uses = instead of ≤; the boundary is a hyperplane with normal vector (1, ..., 1)... figure shows the case with $I = 2$



$\left\{(u_1,u_2): \sum_{i=1}^{I} u_i \le \sum_{i=1}^{I}\phi_i(\bar{x}_i) - \sum_{j=1}^{J} c_j(\bar{q}_j) + \boldsymbol{\omega}_m\right\}$

$\left\{(u_1,u_2): \sum_{i=1}^{I} u_i \le \sum_{i=1}^{I}\phi_i(x_i^*) - \sum_{j=1}^{J} c_j(q_j^*) + \boldsymbol{\omega}_m\right\}$

The right side is fixed so if we differentiate we get $\sum_{i=1}^{I} du_i = 0$; in the case of 2 consumers, we get $du_1 + du_2 = 0 \Rightarrow du_1/du_2 = -1$

Since each x_i is fixed, moving along the frontier corresponds to changing the amount of numeraire

Pareto Optimal Allocation - picking different allocations $(\bar{\mathbf{x}},\bar{\mathbf{q}})$, moves the conditional utility possibilities frontier; we want to move it out as far as possible while maintaining a feasible allocation; i.e., choose $\mathbf{x}$ and $\mathbf{q} \ge \mathbf{0}$ in order to

Max $\sum_{i=1}^{I}\phi_i(x_i) - \sum_{j=1}^{J} c_j(q_j) + \boldsymbol{\omega}_m$ s.t. $\sum_{i=1}^{I} x_i = \sum_{j=1}^{J} q_j$

Note: we can leave of the ω_m since it's a constant, but realize that changing ω_m will change to level of total utility (but not the optimal $\mathbf{x}^*$, $\mathbf{q}^*$... similar to the result we found for CE on p.4)

Marshallian Aggregate Surplus - $\sum_{i=1}^{I}\phi_i(x_i) - \sum_{j=1}^{J} c_j(q_j)$; total utility generated from consumption of good ℓ less its costs of production (in terms of the numeraire); area under demand curve minus area under supply curve... we'll prove this later

Solution - form lagrangian: $L = \sum_{i=1}^{I}\phi_i(x_i) - \sum_{j=1}^{J} c_j(q_j) - \mu\left(\sum_{i=1}^{I} x_i - \sum_{j=1}^{J} q_j\right)$

Don't need to worry about second order conditions because we already made assumptions to guarantee them (see p.3-4)

K-T Conditions - written out on single line (like we did on p.4)

$L_{x_i} = \phi_i'(x_i^*) - \mu \le 0$, with equality of $x_i^* > 0$, $\forall\ i = 1, \ldots, I$

$L_{q_j} = -c_j'(q_j^*) - \mu \le 0$, with equality of $q_j^* > 0$, $\forall\ j = 1, \ldots, J$

$$-L_\mu = \sum_{i=1}^{I} x_i - \sum_{j=1}^{J} q_j = 0$$

*** If $\mu = p^*$ these conditions are exactly the same as the CE conditions ***

First Fundamental Theorem of Welfare Economics - if allocation $(x_1^*, \ldots, x_I^*, q_1^*, \ldots, q_J^*)$ and price p^* constitute a competitive equilibrium, then this allocation is Pareto optimal

Example - 2 consumers and 1 firm; assume quasi-linear utility

Assumptions - $u_i = m_i + \phi_i(x_i)$, $i = 1,2$

$c = c(x_1 + x_2)$... this imposes feasibility because $q = x_1 + x_2$

ω_m = endowed numeraire

Take any feasible allocation $(\bar{x}_1, \bar{x}_2)$

3 Things Hold - (1) $u_1 = m_1 + \phi_1(\bar{x}_1)$

(2) $u_2 = m_2 + \phi_2(\bar{x}_2)$

(3) $m_1 + m_2 = \omega_m - c(\bar{x}_1 + \bar{x}_2)$

The third one says the amount of numeraire consumed = total numeraire endowed to society minus numeraire used for production

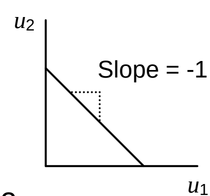
Solve for u_1 as function of u_2 -

Substitute for m_1 from (3) into (1): $u_1 = [\omega_m - c(\bar{x}_1 + \bar{x}_2) - m_2] + \phi_1(\bar{x}_1)$

Substitute for m_2 from (2) into this: $u_1 = [\omega_m - c(\bar{x}_1 + \bar{x}_2) - (u_2 - \phi_2(\bar{x}_2))] + \phi_1(\bar{x}_1)$

Collect terms - $u_1 = -u_2 + [\omega_m + \phi_1(\bar{x}_1) + \phi_2(\bar{x}_2) - c(\bar{x}_1 + \bar{x}_2)]$

This defines the utility possibilities frontier... note that $\partial u_1/\partial u_2 = -1$



No quasi-linear utility - modify example with Cobb-Douglas utility: $u_i = m_i\phi_i(x_i)$, $i = 1,2$

Take any feasible allocation $(\bar{x}_1, \bar{x}_2)$

3 Things Hold - (1) $u_1 = m_1\phi_1(\bar{x}_1)$

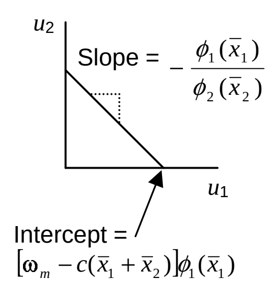
(2) $u_2 = m_2\phi_2(\bar{x}_2)$

(3) $m_1 + m_2 = \omega_m - c(\bar{x}_1 + \bar{x}_2)$

Solve for u_1 as function of u_2 -

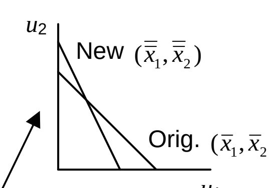
Substitute for m_1 from (3) into (1): $u_1 = [\omega_m - c(\bar{x}_1 + \bar{x}_2) - m_2]\phi_1(\bar{x}_1)$

Substitute for m_2 from (2) into this: $u_1 = \left[\omega_m - c(\bar{x}_1 + \bar{x}_2) - \left(\frac{u_2}{\phi_2(\bar{x}_2)}\right)\right]\phi_1(\bar{x}_1)$



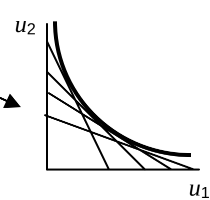
Collect terms - $u_1 = -\frac{\phi_1(\bar{x}_1)}{\phi_2(\bar{x}_2)}u_2 + [\omega_m - c(\bar{x}_1 + \bar{x}_2)]\phi_1(\bar{x}_1)$

This defines the utility possibilities frontier... note that $\partial u_1/\partial u_2 = -\frac{\phi_1(\bar{x}_1)}{\phi_2(\bar{x}_2)}$



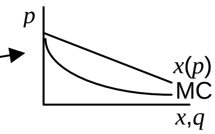
Pick another feasible allocation $(\bar{\bar{x}}_1, \bar{\bar{x}}_2)$ ... slope and intercept of frontier change

Repeat for all feasible allocations and get convex utilities possibilities frontier



Finding PE allocation isn't as easy as just maximizing Marshallian aggregate surplus because $\mathbf{x}$ is changing

Failures of First FTWE - 3 broad categories where markets are not efficient; these are considered **market failures**; won't study all of these in this class



1. Existence Problem - economics of scale... will always have p > MC
2. Market Power - not all agents are price takes; can result from monopolies and oligopolies; also results from market intervention (taxes and price controls)
3. Incomplete Markets - externalities, lack of property rights, public goods

Relating p* and μ -

Marginal Social Benefit - any consumer consuming good ℓ (i.e., $x_i > 0$) will have additional benefit $\phi_i'(x_i) = \mu$; others have $\phi_i'(0) < \mu$ (value of first unit below μ)

Marginal Social Cost - any firm producing good ℓ (i.e., $q_j > 0$) will have marginal cost $c_j'(q_j) = \mu$; others have $c_j'(0) > \mu$ (value of first unit above μ)

Equilibrium - MSB = MSC

Price Signals - forces consumers to consume right amount and producers to produce right amount

Exercise - first order conditions imply that $\phi_i'(x_i) = \phi_j'(x_j) > \phi_k'(x_k)$ for any two consumers i and j who consume good ℓ and any consumer k who does not consume. Show that if this doesn't hold, a Pareto improvement is possible.

Assume consumers i and j consume good ℓ (i.e., x_i & $x_j > 0$)

Without loss of generality assume $\phi_i'(x_i) > \phi_j'(x_j)$

Now consumer i can pay consumer j $\phi_j'(x_j)$ for 1 unit of good ℓ; consumer j would be just as well off and consumer i would be better off by $\phi_i'(x_i) - \phi_j'(x_j) > 0$

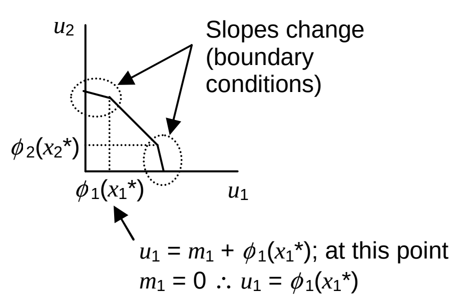
Exercise - first order conditions imply that $c_j'(q_j) = c_k'(q_k) < c_z'(q_z)$ for any two firms j and k which produce good ℓ and any firm z which does not produce. Show that if this doesn't hold, a Pareto improvement is possible... argument is practically identical to the previous one.

Second Fundamental Theorem of Welfare Economics - for any Pareto optimal levels of utility $(u_1^*, ..., u_I^*)$, there are transfers of the numeraire commodity $(T_1, ..., T_I)$ satisfying $\sum_{i=1}^{I} T_i = 0$, such that a competitive equilibrium reached from the endowments $(\omega_{m1} + T_1, ..., \omega_{mI} + T_I)$ yields precisely the utilities $(u_1^*, ..., u_I^*)$

This essentially the converse of the first FTWE; each PO point can be achieved by reallocating the numeraire and finding the corresponding CE

For two-good, quasilinear utility model $\mathbf{x}^*$ and $\mathbf{q}^*$ are independent of the allocation of ω_m

Restricting m_i - if we restrict $m_i \geq 0$, we are basically enforcing an income constraint; this brings up problems with boundary conditions... we don't want that complication to detracts from the welfare discussion so we leave m_i unrestricted



Welfare Analysis

Why - assume we're at a point that's not efficient; if we enact some policy change (or there's some other change in the economy), we want to be able toe measure the effect on social welfare

Marshallian Welfare - $S \equiv$ sum of all consumer utilities minus sum of all producer costs; area under the demand curve minus the area under the supply curve (i.e., area between the demand and supply curves)

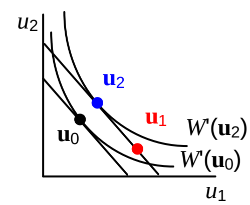
Quasi-Linear Utility - for our model with quasi-linear utility $S \equiv \sum_{i=1}^{I} \phi_i(x_i) - \sum_{j=1}^{J} c_j(q_j)$

Assumptions - if these 2 hold, changes in social welfare can be measured by changes in Marshallian Welfare

(i) **Welfare Function** - social welfare can be measured by some social welfare function $W = W(u_1, ..., u_I)$ that is increasing in any u_i (i.e., $\partial W/\partial u_i > 0$)

Paretian Criteria - $u_i \uparrow \Rightarrow \Delta u_j \geq 0$ (i.e., if one person improves, nobody else is made worse off); this is satisfied by W being increasing in each u_i

(ii) **Redistribution** - there is some central authority who redistributes wealth by means of transfers of the numeraire commodity in order to maximize social welfare



Interpretation - the first assumption says we can tell if some policy moves the utility possibilities curve outward (say from $\mathbf{u}_0 = (u_{1,0}, u_{2,0})$ to $\mathbf{u}_1$); the second says that once we're on the better curve, we can move to any point on that curve to ensure social welfare is improved (from $\mathbf{u}_1$ to $\mathbf{u}_2$)

Area Between Curves - for simplicity take any feasible solution with all consumers consuming a positive amount ($x_i > 0$) and all firms producing a positive amount ($q_j > 0$); under the following conditions S = area between demand and supply curves

(i) $\phi_i'(x_i) = \phi_k'(x_k) \ \forall \ i,k$ (i.e., output allocated efficiently)

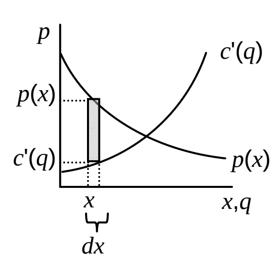
(ii) $c_j'(q_j) = c_k'(q_k) \ \forall \ j,k$ (i.e., output produced efficiently)

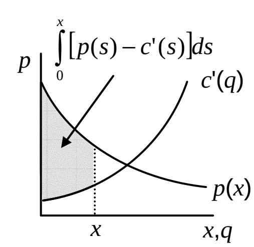
These conditions are fairly realistic and almost always hold (even with monopoly, oligopoly, taxes, etc.)

Implications of Assumptions -

(a) if (i) holds we can say $p(x) \equiv \phi_i'(x_i)$ (i.e., inverse demand = marginal social benefit)

(b) if (ii) holds we can say $c'(q) \equiv c_j'(q_j)$ (i.e., inverse supply = marginal social cost)





Differentiate $S \equiv \sum_{i=1}^{I} \phi_i(x_i) - \sum_{j=1}^{J} c_j(q_j)$

$$dS = \sum_{i=1}^{I} \phi_i'(x_i)dx_i - \sum_{j=1}^{J} c_j'(q_j)dq_j$$

All $\phi_i'(x_i)$ are the same and equal to $p(x)$; all $c_j'(q_j)$ are the same and equal to $c'(q)$:

$$dS = p(x)\sum_{i=1}^{I} dx_i - c'(q)\sum_{j=1}^{J} dq_j$$

Let $\sum_{i=1}^{I} dx_i = dx$ (change in total consumption) and $\sum_{j=1}^{J} dq_j = dq$ (change in total production); since we assumed a feasible allocation we must have $x = q$ and $dx = dq$

$$dS = [p(x) - c'(x)]dx$$

By the magic of integral calculus

$$S = \int_0^x dS = \int_0^x [p(s) - c'(s)]ds = S(x) - S(0)$$

Long-Run Competitive Equilibria

Key Difference - number of firms is endogenous

Free Entry - assume infinite supply of equally efficient firms who could potentially enter the market; firms enter if they can make positive profits at the market price

Free Exit - no fixed costs in long-run so firms can make zero profit by producing zero output ($c(0) = 0$); firms leave if they make negative profits at the market price

Entry-Exit Equilibrium - in long-run equilibrium number of firms doesn't change so firms make zero profit

Long-Run Analysis - trying to find price (p), output for each firm (q), and number of firms (J); total market output is $Q = Jq$

Competitive Equilibrium (CE) - (p^*, q^*, J^*) that satisfies three conditions:

1. <u>Profit Maximization</u> - q^* solves Max $\pi = p^*q - c(q)$ s.t. $q \geq 0$
2. <u>Demand = Supply</u> - $x(p^*) = J^*q^*$
3. <u>Free Entry (zero profit)</u> - $p^*q^* - c(q^*) = 0$

Long-Run Supply Correspondence -

$$Q(p) = \begin{cases} \infty & \pi(p) > 0 \\ \{Q \geq 0 : Q = Jq \text{ for some integer } J \geq 0 \text{ and } q \in q(p)\} & \pi(p) = 0 \end{cases}$$

Existence Problem - since J must be an integer, there could be existence problems

Constant Returns to Scale - in this case there is no existence problem

Increasing Returns to Scale - no long-run competitive equilibrium can exist

Welfare - if there isn't an existence problem the First FTWE holds; the second may or may not hold (again, because of existence problem)

Monopoly

(Tirole Chpt 1 & Section 2.2)

Topics to cover...
- Basics of single product monopoly... done
- Multi-product monopoly... almost
  - Peak-load pricing example... ???
- Cost distortions (produce inefficiently?)... ???
- Rent seeking... done
  - R&D example... done
- Durable goods monopoly... done
- Product selection by monopolist (section 2.2)... done

Single Product Monopoly

Assumptions -
1. Single firm selling product of known quality
2. Monopolist doesn't price discriminate (i.e., charges same price for each good produced)

Monopoly Power - firm can raise its price above marginal cost without losing all its clients; leads to price that is too high resulting in "dead-weight" welfare loss

Other Problems -
- **Costs** - theory & practice suggest it's more difficult for monopoly to keep control over its costs (vs. firms in competitive markets)
- **Rent Seeking** - contests among firms to obtain or secure monopoly power may involve socially wasteful expenditures which partially (or totally) dissipate monopoly profit

Notation -
- **Price** - $p = P(q)$ [inverse demand function with $P'(q) < 0$]
- **Quantity** - $q = D(p)$ [demand function with $D'(p) < 0$]
- **Cost** - $C(q)$ with $C'(q) > 0$ (increasing marginal cost)

Maximize Profit - profit-maximizing monopolist chooses monopoly price p^m in order to maximize profit which equals total revenue (price times quantity) minus total cost (as function of quantity $D(p)$); we'll assume second order conditions are satisfied

$$\text{Max } \pi(p) = pD(p) - C(D(p))$$

1st Order Condition - partial wrt p; first term requires product rule; second term requires chain rule

$$\pi' = D(p) + D'(p)p - C'(D(p))D'(p) = 0$$

Optimal Price Markup - profit margin; price minus marginal cost: $p^m - C' = \dfrac{-D}{D'}$

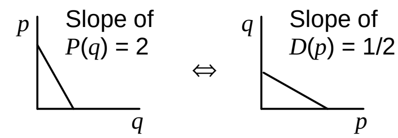
Inverse Elasticity Rule - recall elasticity of demand: $\boxed{\varepsilon = \dfrac{-D'p}{D}}$

Lerner Index - relative markup; ratio between profit margin and price; inversely proportional to the demand elasticity

$$\boxed{\frac{p^m - C'}{p^m} = \frac{-D}{D'p^m} = \frac{1}{\varepsilon}}$$

<u>Exercise</u> - use $p^m - C' = \frac{-D}{D'}$ to show MC = MR

Solve for C' (marginal cost): $C' = p^m + \frac{D}{D'}$



Realize $q^m = D(p^m)$: $C' = p^m + \frac{q^m}{D'}$

Because of the inverse relationship between $D(p)$ and $P(q)$, the slopes (i.e., first derivatives) of these functions are inverses of each other (i.e., $P' = 1/D'$)

That means $C' = p^m + q^m P'$ or MC = MB

Other Method - can look at profit as function of quantity:

$$\text{Max } \pi(q) = P(q)q - C(q)$$

1st Order Condition - partial wrt q; first term requires product rule

$$\partial\pi / \partial q = P'(q)q + P(q) - C'(q) = 0$$

MC = MB - 1st order condition says marginal revenue = marginal cost

MR = price of the next unit ($P(q)$) plus the change in revenues resulting from the new price ($P'(q)q$)

Lesson - there are two ways to represent demand (and supply); sometimes it's easier to derive results with one of them so if you get stuck, try the other method

Elastic Demand - monopoly will always operate in price region such that elasticity of demand exceeds 1; if $\varepsilon < 1$, $p\uparrow \Rightarrow$ TR$\uparrow$; also, since $C' > 0$, $p\uparrow$ (i.e., $q\downarrow$) $\Rightarrow$ TC$\downarrow$ $\therefore$ $\pi\uparrow$

Lower Cost ⇒ Lower Price - monopoly price is a nondecreasing function of marginal cost; monopoly with "unambiguously lower cost" will charge a lower (or the same) price

<u>Proof</u>: suppose $C_1(q) < C_2(q)$ because $C_1'(q) < C_2'(q)\ \forall\ q$ and fixed costs are the same

=Because p_1^m and q_1^m are optimal for $C_1(q)$, we know

$p_1^m q_1^m - C_1(q_1^m) \geq p_2^m q_2^m - C_1(q_2^m)$

Also p_2^m and q_2^m are optimal for $C_2(q)$, so $p_2^m q_2^m - C_2(q_2^m) \geq p_1^m q_1^m - C_2(q_1^m)$

Add these inequalities together: $-C_1(q_1^m) - C_2(q_2^m) \geq -C_1(q_2^m) - C_2(q_1^m)$

Put it all on the left side: $\left[C_2(q_1^m) - C_2(q_2^m)\right] - \left[C_1(q_1^m) - C_1(q_2^m)\right] \geq 0$

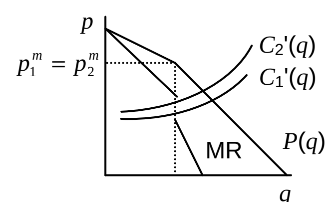
Use fundamental theorem of calculus: $\int_{q_2^m}^{q_1^m} \left[C_2'(x) - C_1'(x)\right] dx \geq 0$

We know $C_1'(q) < C_2'(q)$ so the difference is > 0; $\therefore$ (by some obscure integration rule)

$q_1^m \geq q_2^m$

Since the firms face the same demand curve, that means $p_1^m \leq p_2^m$

Case where $p_1^m = p_2^m$ is kinked demand curve as shown in picture

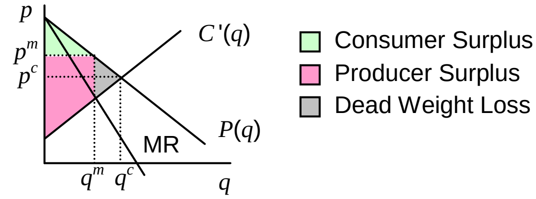


Dead Weight Loss - Lerner Index shows how price changes, but what we're really concerned with is the change in social welfare; monopolist increases price above the perfectly competitive price which results in equilibrium that doesn't maximize Marshallian welfare

Inefficient - maximizing Marshallian welfare requires $p = C'(q)$; if monopolist produces here,

$$\frac{\partial \pi}{\partial q} = P'(q)q + P(q) - C'(q) = P'(q)q < 0$$

$\therefore$ firm can make more π by decreasing q (increasing price)



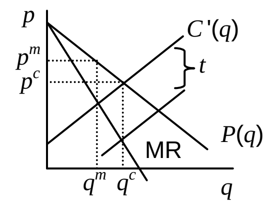
Taxing/Subsidizing Monopolist - to get to optimal q, let t denote the per unit tax ($t < 0$ would be a subsidy); p denote the take home price of the monopolist; $p + t$ is the price to consumers

$$\text{Max } \pi(p) = pD(p + t) - C(D(p + t))$$

Efficiency - requires that $p + t = C'(q)$

1st Order Condition - partial wrt p; first term requires product rule; second term requires chain rule

$$\pi' = D(p+t) + D'(p)p - C'(D(p+t))D'(p+t) = 0$$



Collect terms: $D + D'(p - C') = 0$

Add and subtract tD': $D - tD' + D'(p + t - C') = 0$

Last term drops out because $p + t = C' \;\therefore\; D - tD' = 0 \;\Rightarrow\; t = \dfrac{D}{D'}$

Another Way - we don't need the fancy $\pm\, tD'$, just substitute $p + t$ for C'

$$D + D'(p - (p+t)) = 0 \;\Rightarrow\; D - D't = 0 \;\Rightarrow\; t = D/D'$$

Subsidy - $D/D' < 0$ $\therefore$ to reach efficient level of output, the monopolist must be subsidized

Difficult to Implement - need to know D/D' at (p^c, q^c) $\therefore$ need to know $D(p)$ and $C'(p)$

Win or Lose? - consumers ultimately pay for the subsidy so we should look at whether social welfare is improved by the subsidy; let $p^m(t)$ denote equilibrium price to consumers with the tax/subsidy t and calculate social welfare $SW(t)$

$$SW(t) = \left[\underbrace{\int_{p^m(t)}^{\infty} D(p)dp}_{\text{Consumer surplus}} + \underbrace{tD(p^m(t))}_{\text{Tax benefit (or subsidy cost) to consumers}}\right] + \left[\underbrace{p^m(t)D(p^m(t))}_{\text{Total revenue}} - \underbrace{C(D(p^m(t)))}_{\text{Total cost}} - \underbrace{tD(p^m(t))}_{\text{Tax cost (or subsidy benefit) to monopolist}}\right]$$

We can simplify the math by letting $\pi^m(p^m(t)) = p^m(t)D(p^m(t)) - C(D(p^m(t)))$; also from here on, we'll drop the arguments for the functions

Note: $p^{m}{}' > 0$; firm with lower marginal cost will charge lower (or same) price (we proved that on previous page); t raises cost so $t\uparrow \Rightarrow p^m\uparrow$

Assumption - no distortion between collecting and distributing tax

Differentiate wrt *t* - do it piece by piece for slow folks like me:

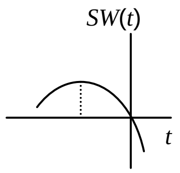
$$\partial\left(\int_{p^m(t)}^{\infty} D(p)dp\right)/\partial t = \int_{p^m}^{\infty} \frac{\partial D}{\partial t}dp + \frac{\partial \infty}{\partial t}D(\infty) - \frac{\partial p^m}{\partial t}D(p^m) = -p^{m}{}'D$$ (using Leibnitz Rule [see end of notes]; notice $\partial D/\partial t = \partial\infty/\partial t = D(\infty) = 0$ so those terms drop out)

$$\frac{\partial\left(tD(p^m(t))\right)}{\partial t} = D + tD'\,p^{m}{'}$$ (using product rule for tD and chain rule for $D(p^m)$)

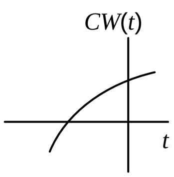
$$\frac{\partial\left(p^m(t)D(p^m(t))\right)}{\partial t} = p^{m}{'}D + p^m D'\,p^{m}{'}$$ (product rule for p^mD and chain rule for $D(p^m)$)

$$SW' = \left[-p^{m}{'}D + D + tD'\,p^{m}{'}\right] + \left[\boldsymbol{\pi}^{m}{'}\,p^{m}{'} - D - tD'\,p^{m}{'}\right]$$

Results -

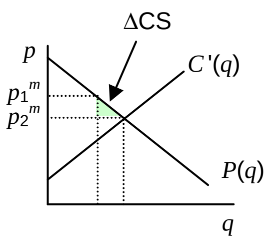


1. $SW'(0) = \left[-p^{m}{'}D + D\right] + \left[\boldsymbol{\pi}^{m}{'}\,p^{m}{'} - D\right] = -p^{m}{'}D < 0$ (because $\pi^{m}{'}(0) = 0$ since monopolist is maximizing profit); this means $t\downarrow \Rightarrow SW\uparrow$ from $t = 0$ so it's optimal to have t < 0 (which we already proved by enforcing the efficiency condition on the monopolist's profit maximization problem on the previous page)



When $p^{m}{'} < 1$, $CW(t)$ is strictly increasing when $t \le 0$

2. Focus on consumer welfare... $CW' = -p^{m}{'}D + D + tD'\,p^{m}{'} = D(1 - p^{m}{'}) + tD'\,p^{m}{'}$
   - $tD'\,p^{m}{'}$ is how CW changes because of subsidy paid for by consumer (or tax paid by monopolist); if $t < 0$ then $tD'\,p^{m}{'} > 0$, so $CW\uparrow$ if $t\uparrow$ (i.e., consumers would prefer not to subsidize the monopolist just based on this term)
   - $D(1 - p^{m}{'})$ is how CW changes because of monopolist's new price (versus monopolist that isn't taxed/subsidized); if $p^{m}{'} < 1$ then this term is > 0 (i.e.,, consumers would prefer not to subsidize the monopolist just based on this term)
   - $\therefore$ if $t < 0$ and $p^{m}{'} < 1$, consumers want $t > 0$ (consumers are better of if we don't subsidize the monopolist, even though subsidizing is more efficient from a social welfare perspective... the gains are enjoyed by the monopolist)



3. $p^{m}{'} < 1$ means for each \$1 of subsidy given to the monopolist, price declines by less than \$1; as we just showed, consumers don't want to subsidize monopolist unless they get full pass along ($p^{m}{'} = 1$)
4. ΔCS (or CW) from $p^m\downarrow$ is second order small (involves two changes); at the margin it goes away

Multiproduct Monopoly

Assumptions -

1. Single firm selling goods $i = 1, ..., n$, charging prices $\mathbf{p} = (p_1, ..., p_n)$, selling quantities $\mathbf{q} = (q_1, ..., q_n)$, where $q_i = D_i(p)$ is the demand for good i
2. Cost of producing q is $C(\mathbf{q}) \neq \sum_{i=1}^{n} C_i(q_i)$, unless there is **cost separability** (where total costs can be decomposed in n subcosts)

Maximize Profit - choose monopoly price $\mathbf{p}^m$ in order to maximize profit; we'll assume second order conditions are satisfied

$$\text{Max } \boldsymbol{\pi}(\mathbf{p}) = \sum_{i=1}^{n} p_i D_i(\mathbf{p}) - C(D_1(\mathbf{p}), \ldots, D_n(\mathbf{p}))$$

1st Order Condition - partial wrt p_i; first term requires product rule and chain rule; second term requires chain rule

$$\boldsymbol{\pi}' = D_i + \sum_{j=1}^{n} p_j \frac{\partial D_j}{\partial p_i} - \sum_{j=1}^{n} \frac{\partial C}{\partial q_j} \frac{\partial D_j}{\partial p_i} = 0$$

Break up summations to separate terms with only good i:

$$\boldsymbol{\pi}' = D_i + p_i \frac{\partial D_i}{\partial p_i} + \sum_{j \neq i} p_j \frac{\partial D_j}{\partial p_i} - \frac{\partial C}{\partial q_i} \frac{\partial D_i}{\partial p_i} - \sum_{j \neq i} \frac{\partial C}{\partial q_j} \frac{\partial D_j}{\partial p_i} = 0$$

Combine the sums:

$$\boldsymbol{\pi}' = \left[D_i + p_i \frac{\partial D_i}{\partial p_i} - \frac{\partial C}{\partial q_i} \frac{\partial D_i}{\partial p_i} \right] + \left[\sum_{j \neq i} \left(p_j - \frac{\partial C}{\partial q_j} \right) \frac{\partial D_j}{\partial p_i} \right] = 0$$

First bracketed term is the usual effect for a single product monopoly; second bracketed term is the **cross effect**: marginal profit of good j times change in demand for good j when price of good i changes

Just for fun, let's don some "algebraic manipulation"; we're trying to get the formula to look something like the Lerner Index (p.1); move everything other than $(p_i - \partial C/\partial q_i)\partial D_i/\partial p_i$ to the right side:

$$\left(p_i - \frac{\partial C}{\partial q_i} \right) \frac{\partial D_i}{\partial p_i} = -D_i - \sum_{j \neq i} \left(p_j - \frac{\partial C}{\partial q_j} \right) \frac{\partial D_j}{\partial p_i}$$

Divide both sides by $\partial D_i/\partial p_i$

$$\left(p_i - \frac{\partial C}{\partial q_j} \right) = -D_i \frac{\partial p_i}{\partial D_i} - \sum_{j \neq i} \left(p_j - \frac{\partial C}{\partial q_j} \right) \frac{\partial D_j}{\partial p_i} \frac{\partial p_i}{\partial D_i}$$

Divide both sides by p_i

$$\frac{(p_i - \partial C / \partial q_i)}{p_i} = -\frac{D_i}{p_i} \frac{\partial p_i}{\partial D_i} - \sum_{j \neq i} \frac{(p_j - \partial C / \partial q_j)}{p_i} \frac{\partial D_j}{\partial p_i} \frac{\partial p_i}{\partial D_i}$$

We now have the left side the way we want it; we need to somehow manipulate the right side to get elasticity of demand; the trick is to multiply it by $p_i D_i D_j / p_i D_i D_j = 1$

$$\frac{(p_i - \partial C / \partial q_i)}{p_i} = \frac{1}{\varepsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - \partial C / \partial q_j)}{p_i} \frac{\partial D_j}{\partial p_i} \frac{\partial p_i}{\partial D_i} \frac{p_i D_i D_j}{p_i D_i D_j}$$

Rearrange the terms so they look like this:

$$\frac{(p_i - \partial C / \partial q_i)}{p_i} = \frac{1}{\varepsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - \partial C / \partial q_j) D_j}{p_i D_i} \left(\frac{\partial D_j}{\partial p_i} \frac{p_i}{D_j} \right) \left(\frac{\partial p_i}{\partial D_i} \frac{D_i}{p_i} \right)$$

Now we can simplify by noticing the elasticity formulas and the fact that $R_i \equiv p_i D_i$ (revenue from product i)

$$\boxed{\frac{(p_i - \partial C / \partial q_i)}{p_i} = \frac{1}{\varepsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - \partial C / \partial q_j) D_j \varepsilon_{ij}}{R_i \varepsilon_{ii}}}$$

Elasticities of Demand -

Own Price - $\varepsilon_{ii} \equiv \frac{-\partial D_i}{\partial p_i} \frac{p_i}{D_i}$ > 0

Cross Price - $\varepsilon_{ij} \equiv \frac{-\partial D_j}{\partial p_i} \frac{p_i}{D_j}$

Substitute - $p_i\uparrow \Rightarrow D_j\uparrow \;\therefore\; \partial D_j/\partial p_i > 0$ and $\varepsilon_{ij} < 0$

Complements - $p_i\uparrow \Rightarrow D_j\downarrow \;\therefore\; \partial D_j/\partial p_i < 0$ and $\varepsilon_{ij} > 0$

Natural Benchmark - n independent monopolists; assume no economies or diseconomies of scope (i.e., no cost effects of producing; single firm's costs are same as independent firms' so we have separable costs)

$C(\mathbf{q}) = \sum_{i=1}^{n} C_i(q_i)$, which simplifies the modified Lerner Index:

$$\frac{(p_i - C_i')}{p_i} = \frac{1}{\varepsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \varepsilon_{ij}}{R_i \varepsilon_{ii}}$$

Only difference from single product monopoly is this term; $\therefore$ difference in pm between multiproduct monopoly and single product monopoly depends on sign of this term.

Substitutes - we know p_j - c_j' is positive, D_j is positive, R_i is positive, and ε_{ii} is positive; $\therefore$ sign of second term depends on ε_{ij}; if we assume all n products are substitutes (for simplicity to avoid ambiguous terms), we have $\varepsilon_{ij} < 0$ which cancels with the minus sign and we end up adding something to $1/\varepsilon_{ii}$; that means the multiproduct firm's optimal price is <u>higher</u> than a single product monopoly's; the single product monopoly's are all competing; they ignore the cross effects and end up charging prices that are to low and don't maximize joint profits

Complements - if we assume all n products are complements, we have $\varepsilon_{ij} > 0$ which means the term subtracted from $1/\varepsilon_{ii}$ is positive so the multiproduct firm's optimal price is <u>lower</u> than a single product monopoly's; in this case, the cross term between the products is an externality to the individual firms; they ignore the benefit of further reducing price brings to the monopolist supplying the complementary good; in fact, the multiproduct monopolist may have one or several goods sold <u>below marginal cost</u> (so their Lerner index may be negative) in order to raise demand for other goods sufficiently (e.g., HP printers to drive demand for printer cartridges)

Rent Seeking

Economic Rent - no real distinction between economic rent and economic profit so rent seeking is the same as profit seeking

Rent Seeking - expending resources to get economic profit either (1) to acquire the monopoly rights or position or (2) to maintain it; some of these resources are wasted so they add to the social cost of a monopoly

Ways to Spend - (1) <u>strategic expense</u> (R&D, patent costs, capital investment, erecting barriers to entry); (2) <u>administrative expenses</u> (lobbying, advertising, legal defense)

Types of Resources - <u>financial</u> (auctions, bribes)... boils down to a transfer between the monopolist and the government so there's no real waste; <u>time</u> (lobbying)... resources are in fact wasted

Extreme Case - Posner (1975) developed two axioms under which all monopoly rents should be counted in the costs of monopoly (i.e., the entire producer surplus of the monopolist is expended in rent seeking

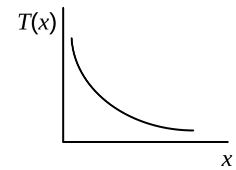
Complete Rent Dissipation - total expenditure by firms to obtain rent is equal to the amount of the rent (e.g., ten firms spend $1 each to obtain 10 percent chance of obtaining $10 profit; total cost is equal to the total profit); this is also called the <u>zero-profit, free-entry condition</u> because it results when there is perfect competition to acquire the monopoly rights

Socially Wasteful Dissipation - expenditure has no socially valuable by-products; looks at use of resources spent not just quantity spent

Real World - Posner's two axioms rarely hold in the real world; although it's almost certain that some fraction of monopoly profits are lost to rent seeking, we can't really determine an amount unless we consider a specific monopoly/industry

R&D Example - not in book; assume first firm to make a discover will have infinitely lived monopoly on the good; monopoly profit per period is π; suppose time is continuous and private and social discount rate is r ($\therefore$ discounted value of payoff is e^{-rt}); discovery is deterministic process (unrealistic) such that spending x dollars on R&D means the discovery date will be $T(x)$

Technical Assumptions -



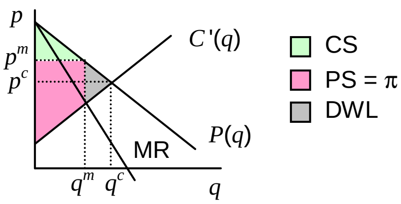
$T' < 0$ - $x\uparrow \Rightarrow T\downarrow$; spend more, means discover quicker)

$T'' > 0$ - diminishing returns to R&D

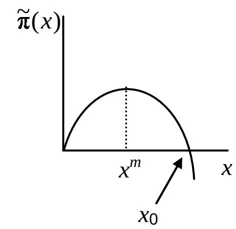
$r(T')^2 - T'' < 0$ - 2nd order condition required for profit function to be concave

$T'(0) = \infty$ and $T(0) = \infty$ - avoids corner solutions

Uncontested Monopoly - only 1 agent has knowledge to seek discovery



$$\text{Max } \tilde{\boldsymbol{\pi}}(x) = \underbrace{\left(\int_{T(x)}^{\infty} \boldsymbol{\pi} e^{-rt}\, dt \right)}_{\text{PV of discovery}} \underbrace{- x}_{\text{R\&D Cost}} = \frac{\boldsymbol{\pi}}{r} e^{-rT(x)} - x$$



1st Order Condition - partial derivative with respect to x:

$\tilde{\boldsymbol{\pi}}' = -\boldsymbol{\pi} T' e^{-rT(x)} - 1 = 0$ ... we can solve for x^m, but don't need to for this discussion

2nd Order Condition - partial derivative again with respect to x:

$\tilde{\boldsymbol{\pi}}'' = \boldsymbol{\pi} r (T')^2 e^{-rT(x)} - \boldsymbol{\pi} T'' e^{-rT(x)} = \boldsymbol{\pi} e^{-rT(x)} \left(r(T')^2 - T'' \right) > 0$ to be concave; it will be because of the assumption we made about $T(x)$

Special Values - x^m is investment in R&D that yields maximum profit; x_0 is investment in R&D that yields zero profit (any $x > x_0$ yields negative profit); this will come in handy when looking at R&D with competition

Competition - assume 2 agents vie for discovery; each with $T(x)$; first to make discovery gets monopoly, other gets nothing (to be complete, if they discover at same time, they flip a coin to award the monopoly); let x_i be amount or R&D expenditure for agent i (i = 1,2); assume sequential moves to pick x_1 and x_2 and only 1 round (Stacklburg)

Subgame Perfect Nash Equilibrium - find agent 2's best reply to any value of x_1:

$$x_2(x_1) = \begin{cases} x^m & x_1 < x^m \text{ (equivalent to agent 1 throwing his money away)} \\ x_1 + \varepsilon & x^m \le x_1 < x0 \text{ (spend tiny bit more; win R\&D race; } \tilde{\boldsymbol{\pi}} > 0\text{)} \\ 0 & x_1 \ge x_0 \text{ (agent 1 wins, doesn't make any profit; } \tilde{\boldsymbol{\pi}}_1 \le 0,\ \tilde{\boldsymbol{\pi}}_2 = 0\text{)} \end{cases}$$

Equilibrium set as 2 points:

(E1) $x_1 = x_0$, $x_2 = 0$ ($\tilde{\boldsymbol{\pi}}_1 = \tilde{\boldsymbol{\pi}}_2 = 0$)

(E2) $x_1 = 0$, $x_2 = x^m$ ($\tilde{\boldsymbol{\pi}}_1 = 0$, $\tilde{\boldsymbol{\pi}}_2 = \tilde{\boldsymbol{\pi}}_{\max}$)

E2 is same as uncontested monopoly; look at social welfare for E1:

$$\left(\int_{T(x_0)}^{\infty} e^{-rt} \text{CS} dt \right) + \tilde{\boldsymbol{\pi}}(x_0)$$

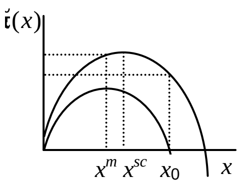
Posner-Like Result - since $\tilde{\boldsymbol{\pi}}(x_0) = 0$, entire producer surplus is expended in R&D race

First Best Social Welfare Optimum - max social welfare with respect to (p,q) [position on demand curve] and x [level of R&D]

Max $\mathrm{SW}(q,x)=\left(\int_{T(x)}^{\infty}[\boldsymbol{\pi}(q)+\mathrm{CS}(q)]e^{-rt}dt\right)-x$ ... has separability; choosing q is independent of x ∴ want to get (p,q) for competitive (efficient) equilibrium

Second Best Social Welfare Optimum - usually more interesting; start with something as given (e.g., monopoly (p,q)) and then maximize social welfare; more realistic from policy point of view; in this case, we assumed monopoly (p,q), need to decide if E1 or E2 is better and then try to create policy to get the better one

Exercise - **E1 or E2?** - given second best welfare problem (i.e., take monopoly pricing as given) is E1 equilibrium better or worse socially than uncontested monopoly (E2)?

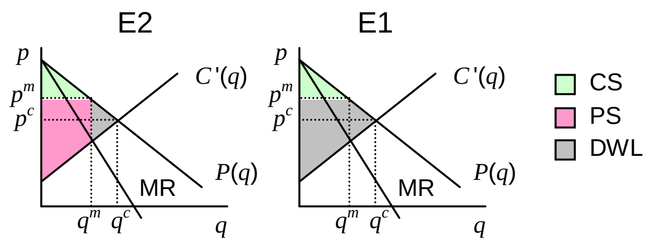


Monopolist makes his decision based on maximizing $\tilde{\boldsymbol{\pi}}(x)=\frac{\boldsymbol{\pi}^m}{r}e^{-rT(x)}-x$, which we derived and graphed earlier. To account for social welfare we have to add consumer surplus back into the equation before integrating and we'd end up with

$$\mathrm{SW}(x)=\frac{\boldsymbol{\pi}^m+\mathrm{CS}^m}{r}e^{-rT(x)}-x$$

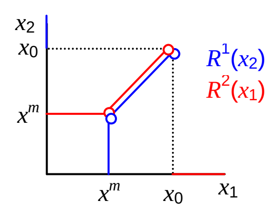
(where CS^m = consumer surplus given monopoly output-price decision). This is a shift in the original curve which moves the peak to the right (i.e., $x^m < x^{sc}$); we need more specific information to determine whether the social welfare is greater under x^m or x_0 (as shown in the graph x^m is better, but we could easily change the parameters to show x_0 being better)

Len's Take - (not necessarily correct) Since there's an infinite time horizon, the time at which the product is developed is irrelevant, only the amount of social welfare available with the product is important ∴ E2 (uncontested monopoly) would be better; if there wasn't an infinite time horizon, we'd have to consider the fact that E1 develops the product more quickly ($x_0 > x^m$); in that case we'd have to compare the total welfare in both cases and pick the better one; regardless of the time horizon, if we look at the problem purely from a consumer's point of view, E1 would always be better



Exercise - **Simultaneous Choice** - same problem with 2 agents, but now they choose R&D level simultaneously

Look at intersection of best reply functions; we showed best reply for second firm earlier; firm 1's best reply would be the same (just swap the 1's and 2's); these functions don't intersect so there is no pure strategy equilibrium

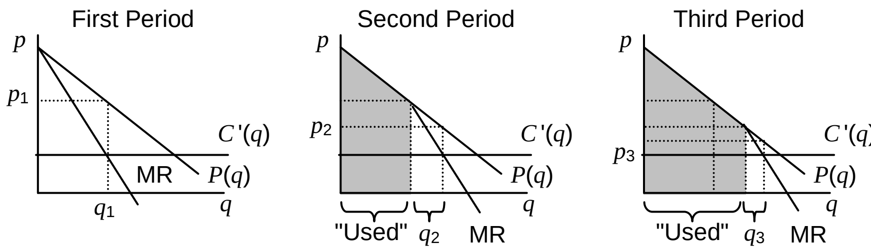


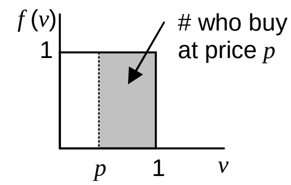
Durable Good Monopoly

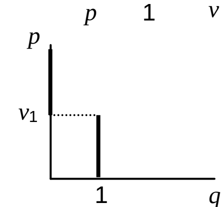
Basic Problem - goods offered by the monopolist at two different dates are substitutes rather than complements; if consumers expect a price decrease and hold back on their purchases, their rational expectations hurt the monopolist

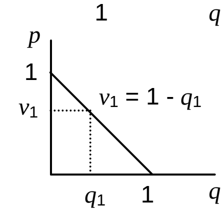
Intertemporal Price Discrimination - demand for services of good is stable; but as more "used" units are available, demand for new units declines over time; effectively, firm ends up price discriminating over time; gives consumers incentive to wait to make purchase decision

Coase Conjecture - as timing between periods declines, monopolist's profits decline (if price changes continuously, p = MC, profit is zero, same as competitive solution); if monopolist can change his price very quickly he will lose his monopoly power completely









Leasing vs. Selling - consider good that lasts two periods with no depreciation

Model of Demand - typical model of demand for services in industrial organization; suppose consumers have value v from using 1 unit of the product and zero value from additional units; if we normalize the population to 1, the population of consumers is characterized by a uniform distribution on v on [0,1].

Individual Demand - if $p > v_1$, consumer doesn't buy; if $p < v_1$, consumer buys (at equally, consumer is indifferent)

Market Demand - $D(p)$ = 1 - $F(p)$ [cumulative distribution]

$$D(p) = 1 - \int_0^p f(v)dv = 1 - \int_0^p 1dv = 1 - p$$

Inverse Demand - $P(q) = 1 - q$

Assumptions - MC = 0; shared discount factor $\delta = \dfrac{1}{1+r}$

Leaser's Optimum - benchmark monopoly optimum; since goods don't stay on the market in the second period, the monopolist faces the exact same demand curve, ∴ each period's optimal rental price solves

$$\text{Max } \pi_t = p_t D(p_t),\ t = 1,2$$

Each period is the same with $\pi = p(1-p) = p - p^2$

1st Order Condition - $1 - 2p = 0 \;\Rightarrow\; p = \dfrac{1}{2} \ldots \therefore\; q = 1 - p = \dfrac{1}{2}$

PV of Profits - $p_1 q_1 + \delta p_2 q_1 = \dfrac{1}{2}\dfrac{1}{2} + \delta \dfrac{1}{2}\dfrac{1}{2} = \dfrac{1}{4}(1+\delta)$

(Note: since firm produced q_1 in first period, those same units are available in the second period, so the firm doesn't produce in the second period)

Seller's Optimum - find optimum second period price conditional on any first period outcome q_1; given q_1 demand for purchases in second period (q_2) is given by

$p_2 = 1 - q_1 - q_2$ (supply in second period equals what producer made before plus what it makes in period 2); problem becomes

$$\text{Max } \pi_2 = p_2 q_2 = (1-q_1-q_2)q_2 = q_2 - q_1 q_2 - q_2^2$$

1st Order Condition - $1 - q_1 - 2q_2 = 0 \;\Rightarrow\; q_2(q_1) = \dfrac{1-q_1}{2}$

$$\therefore\; p_2(q_1) = 1 - q_1 - \frac{1-q_1}{2} = \frac{1-q_1}{2}$$

$$\pi_2(q_1) = p_2 q_2 = \left(\frac{1-q_1}{2}\right)^2 = \frac{(1-q_1)^2}{4}$$

First Period - look at person who consumes services in both periods; valuation of marginal consumer (someone who is just willing to buy) in both periods is

$$\underbrace{\underbrace{v\cdot(1+\delta)}_{\text{value of services over 2 periods)}} - \underbrace{p_1}_{\text{cost of buying}}}_{\text{value of services over 2 periods)}} = \underbrace{\delta\cdot(v - p_2^a)}_{\text{CS of buying next period}}$$

(consumer is indifferent between buy now or later)

p_2^a is the anticipated 2nd period price

Solve for $p_1 = v + \delta p_2^a$

Sub $v = 1 - q_1$ (inverse demand)

$p_1 = 1 - q_1 + \delta p_2^a$

Rational Consumer - $p_2^a = p_2 = \dfrac{1-q_1}{2}$; equivalent to subgame perfect equilibrium;

$\therefore$ inverse demand in first period is $p_1 = 1 - q_1 + \delta\dfrac{1-q_1}{2} = (1-q_1)\left(1+\dfrac{\delta}{2}\right)$;

monopolist solves

$$\text{Max } \pi = p_1 q_1 + \delta p_2 q_2 = q_1(1-q_1)\left(1+\frac{\delta}{2}\right) + \delta\frac{(1-q_1)^2}{4}$$

1st Order Condition - $\dfrac{\partial}{\partial q_1}\left[q_1 + \dfrac{\delta}{2}q_1 - q_1^2 - \dfrac{\delta}{2}q_1^2 + \dfrac{\delta}{4}(1-q_1)^2\right] =$

$$1 + \frac{\delta}{2} - 2q_1 - \delta q_1 - \frac{\delta}{2}(1-q_1) = 1 - 2q_1 - \frac{\delta}{2}q_1 = 0 \;\Rightarrow\; q_1 = \frac{2}{(4+\delta)}$$

$$\therefore\; p_1 = \left(1 - \frac{2}{4+\delta}\right)\left(1+\frac{\delta}{2}\right) = 1 - \frac{2}{4+\delta} + \frac{\delta}{2} - \frac{2\delta}{2(4+\delta)} = \frac{4+4\delta+\delta^2}{2(4+\delta)} = \frac{(2+\delta)^2}{2(4+\delta)}$$

$\pi = \dfrac{(2+\delta)^2}{4(4+\delta)}$ (according to Prof Romano's math... I couldn't get it to come out to that)

Comparison - $q_1^s < q_1^l$, $p_1^s < p_1^l$, $q_2^s < q_1^l$ (recall in lease, firm still leases q_1 in second period) and $p_2^s < p_2^l \;\Rightarrow\; \pi^s < \pi^l$ (as long as $\delta > 0$)

Evading Coase Problem -

1. Lease - just showed lease is better, but has it's own problems: transaction costs; moral hazard [mistreatment of goods unless firm invests in monitoring technology]; lack of anonymity [customer's who leased signaled high willingness to pay for good]
2. Credible Commitment - economic agent can always do as well when he can commit himself as when he cannot (because he can always duplicate what he does under no commitment into the commitment); in this case, a commitment would prevent people

from anticipating the price being reduced in the second period; one way to do that is to destroy the production facility (e.g., lithograph molds)

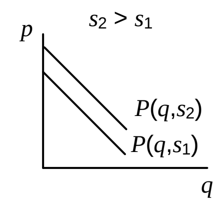
3. Reputation Equilibrium - Disney videos... only released very 7 years to keep price up
4. Rebates - "not on the exam"

Product Selection by Monopolist

Product Quality - original work done by Spence (Bell Journal, 1975)

Basic Model - s = product quality

Inverse Demand - $P(q,s)$ with $P_q < 0$ (marginal benefit of another unit of the product) and $P_s > 0$ (marginal benefit of additional quality)



Cost Function - $C(q,s)$ with $C_q > 0$ (marginal cost of another unit of the product) and $C_s > 0$ (marginal cost of additional quality)

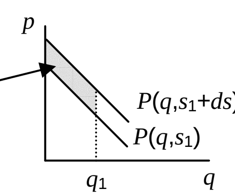
Welfare Maximization - 1st best social welfare optimum; max social welfare with respect to (p,q) [position on demand curve] and s [level of quality]

$$\text{Max } W(q,s) = \underbrace{\left[\int_0^q P(x,s)dx\right]}_{\substack{\text{Area under demand curve} \\ \text{"gross consumer surplus"}}} - C(q,s)$$

1st Order Condition - use Leibnitz Rule; for ∂q, q only enters in upper limit of integral so we only use the green term; for ∂s, s only enters in the function so we only use the pink term [see end of notes for the color codes]

$$\frac{\partial SW}{\partial q} = \frac{\partial q}{\partial q}P(q,s) - C_q = P(q,s) - C_q = 0 \;\Rightarrow\; P(q,s) = C_q$$

$$\frac{\partial SW}{\partial s} = \left[\int_0^q \frac{\partial P(x,s)}{\partial s}dx\right] - C_s = \left[\int_0^q P_s(x,s)dx\right] - C_s = 0 \;\Rightarrow\; \int_0^q P_s(x,s)dx = C_s$$



The first one says price (marginal benefit of an additional unit) is equal to marginal cost (same as before); the second one says the marginal benefit of quality is equal to the marginal cost of quality

Average Marginal Valuation of Quality - divide by q: $\frac{1}{q}\int_0^q P_s(x,s)dx = \frac{C_s}{q}$

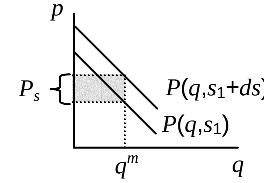
Monopoly - 2nd best social welfare optimum; given that we have a monopoly (i.e., (p,q) is set), what level should s be? first we'll look at the monopoly level:

$$\text{Max } \pi(q,s) = P(q,s)\cdot q - C(q,s)$$

1st Order Condition -

$$\frac{\partial \pi}{\partial q} = P + P_q q - C_q = 0 \;\Rightarrow\; P + P_q q = C_q \quad \text{(MR = MC)}$$

$$\frac{\partial \pi}{\partial s} = P_s q - C_s = 0 \;\Rightarrow\; P_s q = C_s \quad \text{(MR of quality = MC of quality)}$$



P_s = value of increased quality to marginal consumer; multiply it by the number of consumers buying and we get marginal revenue of increased quality

Difference - social planner looks at effect of an increase in quality on *all* consumers; the monopolist considers the effect on the *marginal* consumer

Compare to Social Optimum - find social welfare as function of q and evaluate it at q^m;

we already did that: $s^w(q^m)$ is determined by $\frac{1}{q^m}\int_0^{q^m} P_s(x,s)dx = \frac{C_s}{q^m}$

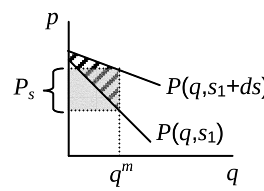
$$\boxed{s^m >, =, < s^w(q^m) \text{ as } \frac{1}{q^m}\int_0^{q^m} P_s(x,s^m)dx \iff P_s(q^m,s^m)}$$

In English - if average MB of quality (the integral term) < MB to marginal consumer (P_s), the monopolist produces <u>too much</u> quality ($s^m > s^w(q^m)$); if average MB of quality > MB to marginal consumer, then monopolist produces <u>too little</u> quality ($s^m < s^w(q^m)$)

2nd Order Condition - this result depends on $W(q, s)$ being single peaked in s... sufficient condition is $P_{ss} \le 0$ and $C_{ss} \ge 0$, at least 1 of these with strict inequality (i.e., W is concave in s)

Examples -

Too Much Quality - look at quality as being <u>information</u> on how to use the product; high value customers are more likely to know how to use the product and don't need this information (e.g., computer software); that means increased in quality is a non-parallel shift of the demand curve (further apart for lower value consumers); since the monopolist decides s based on the marginal consumer, the MB to the marginal consumer is greater than the MB to the average consumer so the monopolists produces too much quality



Same Amount - look at quality as reducing transactions cost of purchases (e.g., more checkout registers); assume transaction cost is same for everyone

p = payment to monopolist for a unit of good

$c(s)$ = transaction cost to consumer of purchasing 1 unit; $c'(s) < 0$ (i.e., monopolist improving quality [s] lowers transaction cost)

$p + c(s)$ = total cost to consumer of 1 unit

$C(q,s)$ = cost to monopolist

$q = D(p + c(s))$... demand function is based on effective cost to consumer

Inverse Demand - $\tilde{p}(q) = D^{-1}(q)$, which in consumer equilibrium (i.e., on the demand curve) also equals $p + c(s)$

Marginal Social Value of q is $p(q,s) = \tilde{p}(q) - c(s)$ (value of q minus transaction cost)

Welfare problem - Max $W(q,s) = \int_0^q P(x,s)dx - C(q,s) = \int_0^q [\tilde{p}(x) - c(s)]dx - C(q,s)$

we can pull $c(s)$ out of integral so $W(q,s) = \int_0^q \tilde{p}(x)dx - c(s)q - C(q,s)$

take monopoly output as given

1st order Condition - $\frac{\partial W}{\partial s} = -c'(s)q^m - C_s = 0$... cost savings for increasing s equals the cost of increasing s

Monopoly problem - Max $\pi = pq - C(q,s)$ such that $\tilde{p}(q) = p + c(s)$ so we really have $\pi = \tilde{p}(q)q - c(s)q - C(q,s)$

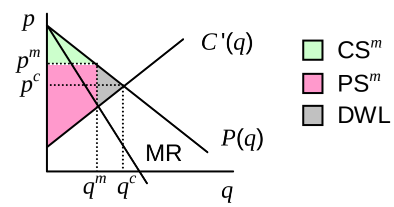
1st order Condition - $\frac{\partial \pi}{\partial s} = -c'(s)q^m - C_s = 0$ ... same as social welfare condition ∴ monopolist produces the optimal level of quality given it's output... <u>this will always be the case when all consumers value quality equally</u> (i.e., average marginal benefit = marginal benefit to marginal consumer)

<u>Exercise</u> - show this result with the general condition

General condition: s^m >, =, < $s^w(q^m)$ as $\frac{1}{q^m}\int_0^{q^m} P_s(x, s^m)dx$ < = > $P_s(q^m, s^m)$

$$\frac{1}{q^m}\int_0^{q^m} P_s(x,s^m)dx = \frac{1}{q^m}\int_0^{q^m} \frac{\partial}{\partial s}\left(\tilde{p}(x) - c(s^m)\right)dx = \frac{1}{q^m}\int_0^{q^m} -c'(s)dx = -c'(s^m)$$

$P_s(q^m, s^m) = \frac{\partial}{\partial s}\left(\tilde{p}(q^m) - c(s^m)\right) = -c'(s^m)$ ... they're the same ∴ $s^m = sw(q^m)$



Number of Products - compare monopolist's product diversity to socially optimal product diversity

Social Value - $PS^m + CS^m$ + DWL (gross surplus); note: $PS^m = \pi^m$ if there are no fixed costs

Nonappropriability - firm generally cannot capture all the gross surplus generated by the product

F = fixed cost related to developing a new product

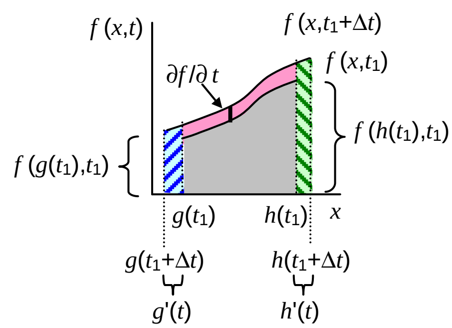
Too Few - if $PS^m + CS^m + DWL > F > PS^m$; product should be introduced, but monopolist won't introduce it; this is 1st best because we're looking at maximum gross surplus, but 2nd best can give same result: $PS^m + CS^m > F > PS^m$

Too Many - multiproduct monopolist could produce too many if the products are substitutes (higher price for 1 good raises demand for other good so PS^m↑), but this is a 1st best phenomena (we can't look at counter effect if monopoly is given)

Leibnitz Rule - differentiating an integral of one variable with respect to another

$$F(t) = \int_{g(t)}^{h(t)} f(x,t)dx$$

$$F'(t) = \underbrace{\int_{g(t)}^{h(t)} \frac{\partial f(x,t)}{\partial t}dx}_{} + \underbrace{h'(t)f(h(t),t)}_{} - \underbrace{g'(t)f(g(t),t)}_{}$$



Price Discrimination

(Tirole Chpt 3)

Topics to cover...
- Intro... done
- Perfect (first degree) PD... done
- Third degree PD... done
  - Positive results... done
  - Normative results... done
- Second degree PD (nonlinear pricing)... done
  - 2 part tariffs (tie-in sales)... done
  - Fully nonlinear tariffs... done
- Quality discrimination... done

Price Discrimination - two units of same good sold at different prices, either to same consumer or to different consumers, and price difference can't be explained solely be cost differences
- **Imperfect Definition** - lots of PD strategies involve varying the product itself (quality)

General Idea - seller can increase profit by price discriminating (unless it's intertemporal like with durable goods); trying to capture consumer surplus

Requirements for Price Discrimination - three things that must be present
- **Market Power** - seller must have market power (barriers to entry) in order to be able to influence price; we'll assume seller is a monopoly
- **No Resale** - resale of the product must be prevented or limited; otherwise those who buy at lower price could become competitor for high value consumers; "produce arbitrage possibility"
  - **Good examples** - services or specialized products (can't be resold)
  - **Bad example** - 2 part tariff (price): $T = A + pq$ (fixed fee plus price per unit times number of units); in this case 1 person could pay A and resell for $p + \delta$... monopoly would loose sales to this consumer
- **ID Consumers** - seller must effect identification of consumers with different willingness to pay
  - **Effect Identification** - could use pricing scheme itself to do it (second-degree) or could identify consumers himself (third-degree... college students, senior citizens, etc.)
  - **Personal Arbitrage** - person of high value posing as low value customer

Types of Price Discrimination - distinguished by Pigou (1920)
- **First Degree** - also called **perfect**; monopoly that can perfectly identify each consumer's value and resale is not possible; not realistic, but good as baseline for potential of price discrimination
- **Second Degree** -
- **Third Degree** - signal of group membership; no resale across groups, but there is within groups (i.e., no further discrimination possible)

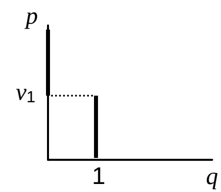
Perfect Price Discrimination

Basics - monopoly; no resale; perfect information about consumers

Constant MC - for this analysis, we'll assume constant marginal cost c (but it's not critical to the results; just simpler math)

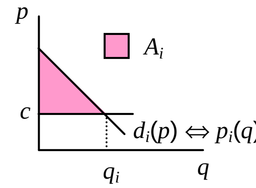
2 Cases of Demand -

Unit Demand - consumers characterized by reservation price (v_i); no utility from additional units; can't buy partial units; individual demand curve is a spike at 1 with height v_i; market demand is characterized by a distribution of reservation prices

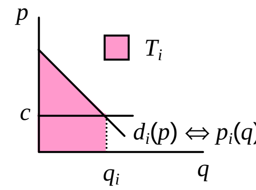


Seller's Optimum - sell to every consumer with $v \geq c$ at personalized price equal to individual's valuation; don't sell if $v < c$

Traditional - each individual consumer has a downward sloping demand curve $d_i(p)$ (inverse demand $p_i(q)$); q_i is quantity at which $p_i(q_i) = c$; there are many pricing schemes that result in perfect discrimination:



Two Part Tariff - don't sell to consumers who have $p_i(0) \leq c$ (valuation below MC); firm sells to all consumers who have $p_i(0) > c$ using personalized two part tariff: $T_i = A_i + cq$, where $A_i = \int_c^{\infty} d_i(p)dp$



Bundle Offer - offer take-it-or-leave-it offer: bundle (T_i,q_i) or nothing, where

$$T_i = \int_0^{q_i} p_i(q)dq$$

Result - both cases result in maximized Marshallian welfare; resulting output level is socially efficient, but all surplus goes to the seller (CS = 0)

Third-Degree Price Discrimination

Basics - m "groups" or markets characterized by some exogenous information (age, sex, occupation, location, etc.); must be some signal that tells the seller which group consumers fall into;

Resale - no resale across (between) groups, but could have resale within a group (i.e., no further discrimination within a group)

** This is a special case of a multi-product monopoly **

Positive Analysis - $q_i = D_i(p_i)$ ... quantity sold in market i = demand for group i at price p_i

$$\max_{p_i} \sum_{i=1}^{m} p_i D_i(p_i) - C\left(\sum_{i=1}^{m} D_i(p_i)\right)$$

FOC - $\frac{\partial \pi}{\partial p_i} = D_i(p_i) + p_i D_i'(p_i) - C'(D_i(p_i))D_i'(p_i) = 0$

Rearrange terms: $p_i - C' = -\frac{D_i(p)}{D_i'(p_i)}$

Divide both sides by p_i : $\frac{p_i - C'}{p_i} = -\frac{D_i(p)}{p_i D_i'(p_i)} = \frac{1}{\varepsilon_i}$ $\left(\varepsilon = -\frac{dq}{dp}\frac{p}{q} = -D'\frac{p}{D}\right)$

Inverse Elasticity Rule - same as regular monopoly; price in each market is based on elasticity; less elastic markets get high price (i.e., high value customers pay higher price; low value customers pay lower price); **note:** if elasticities between markets aren't different, there's not gain to price discriminating

Normative Analysis - welfare analysis

Second Best - we're assuming monopoly; we want to compare no price discrimination versus third-degree price discrimination; would welfare be higher or lower if we made price discrimination illegal?

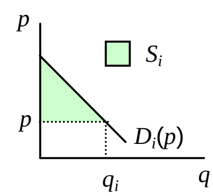
Intuition - social welfare equals producer plus consumer surplus; SW = PS + CS; $PS_3 \geq PS_{ND}$; CS can go ↑ or ↓ depending on p↑ or p↓ so we have uncertain ΔCS which means we have uncertain ΔSW

Formally - originally done by Varian (1985); assume constant returns to scale: $C(q) = cq$

Let $(\overline{p}, \overline{q}_i)$ be non-discriminating price and quantity in group i; $\overline{q}_i = D_i(\overline{p})$

Let (p_i, q_i) be price discriminating price and quantity in group i; $q_i = D_i(p_i)$

Consumer Surplus - $S_i(p) = \int_p^\infty D_i(x)dx$ be consumer surplus for group i with price p



$$S_i'(p) = \frac{d}{dp}\int_p^\infty D_i(x)dx = -D_i(p) < 0 \quad \text{(using Leibnitz Rule)}$$

$$S_i''(p) = \frac{d}{dp}\left[-D_i(p)\right] = -D_i'(p) > 0$$

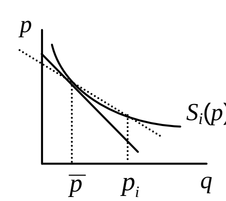
$\therefore$ $S_i(p)$ is <u>decreasing convex</u> function (everywhere above it's tangent)

Non-discriminating social welfare: $\overline{W} = \left[\sum_{i=1}^m S_i(\overline{p})\right] + \left[\sum_{i=1}^m (\overline{p} - c)\overline{q}_i\right]$

Price discriminating social welfare: $W^D = \left[\sum_{i=1}^m S_i(p_i)\right] + \left[\sum_{i=1}^m (p_i - c)q_i\right]$

Change in welfare: $\Delta W = \left[\sum_{i=1}^m \left[S_i(p_i) - S_i(\overline{p})\right]\right] + \left[\sum_{i=1}^m (p_i - c)q_i\right] - \left[\sum_{i=1}^m (\overline{p} - c)\overline{q}_i\right]$

Use convexity of $S_i(p)$: $S_i(p_i) - S_i(\overline{p}_i) \geq S_i'(\overline{p})(p_i - \overline{p})$



<u>Proof</u>: look at $p_i > \overline{p}$; line connecting $(\overline{p}, S_i(\overline{p}))$ and $(p_i, S_i(p_i))$ will be flatter (less negative) than slope of S_i at $\overline{p}$:

$\frac{S_i(p_i) - S_i(\overline{p}_i)}{(p_i - \overline{p})} \geq S_i'(\overline{p})$; multiply both sides by $(p_i - \overline{p})$

look at $p_i < \overline{p}$; line connecting $(\overline{p}, S_i(\overline{p}))$ and $(p_i, S_i(p_i))$ will be steeper (more negative) than slope of S_i at $\overline{p}$:

$\frac{S_i(p_i) - S_i(\overline{p}_i)}{(p_i - \overline{p})} \leq S_i'(\overline{p})$; multiply both sides by $(p_i - \overline{p}) < 0$ so inequality flips

$\therefore$ $\Delta W \geq \left[\sum_{i=1}^m \left[S_i'(\overline{p})(p_i - \overline{p})\right]\right] + \left[\sum_{i=1}^m (p_i - c)q_i\right] - \left[\sum_{i=1}^m (\overline{p} - c)\overline{q}_i\right]$

Substitute $S_i'(\bar{p}) = -D_i(\bar{p}) = -\bar{q}_i$:

$$\Delta W \geq \left[\sum_{i=1}^{m} [\bar{q}_i(\bar{p} - p_i)]\right] + \left[\sum_{i=1}^{m} (p_i - c)q_i\right] - \left[\sum_{i=1}^{m} (\bar{p} - c)\bar{q}_i\right]$$

Combine sums: $\Delta W \geq \sum_{i=1}^{m} [\bar{q}_i \bar{p} - \bar{q}_i p_i + p_i q_i - cq_i - \bar{p}\bar{q}_i + c\bar{q}_i]$

Cancel terms and regroup: $\Delta W \geq \sum_{i=1}^{m} [(p_i - c)(q_i - \bar{q}_i)]$

Sub $\Delta q_i = q_i - \bar{q}_i$: $\boxed{\Delta W \geq \sum_{i=1}^{m} (p_i - c)\Delta q_i}$

We can repeat this derivation expanding around p_i (instead of $\bar{p}$) and get:

$$\boxed{\Delta W \leq (\bar{p} - c)\sum_{i=1}^{m} \Delta q_i}$$

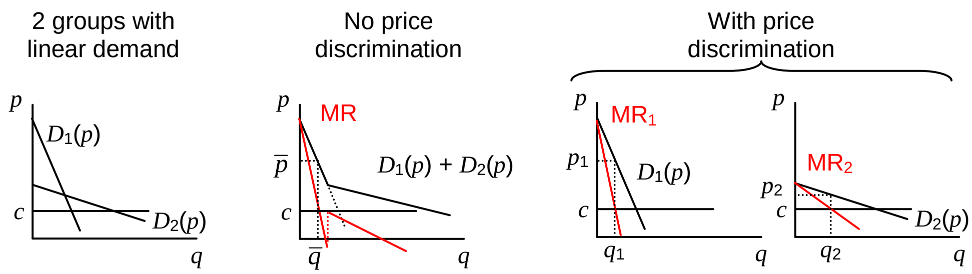
Results -

(a) From lower bound, if all $\Delta q_i \geq 0$, then $\Delta W \geq 0$; improving on original distortion; monopoly causes problem because q too low; price discrimination that increases q (without decreasing in any markets) will improve social welfare (if q_i decreases in some, the result is ambiguous)

(b) From upper bound, if $\sum_{i=1}^{m} \Delta q_i < 0$, then $\Delta W < 0$; if we end up with less output, social welfare goes down

(c) If price discrimination leads to same output with different prices, $\Delta W < 0$; this result doesn't seem intuitive, but consider definition of inefficiency (all consumers have MB = MC so there are no trades that lead to Pareto improvement); in this case, different prices with same total output as single-price monopoly indicates that we can have an improvement by shifting consumption between consumers

(d) Possible to have no other effect than to open new markets; consider case with linear demand below: $q_1 = \bar{q}$ and $p_1 = \bar{p}$, so there's no change to group 1, but a new market is added for group 2



Second-Degree Price Discrimination

Basics - individuals have downward sloping demand (allows non-linear pricing); resale infeasible; seller known distribution of demands, but can't identify individual consumers so he can't prevent "personal arbitrage"

Personal Arbitrage - since seller can't identify customers, they can claim to be different than they are (i.e., high value customer posing as low value to get the lower price)

Non-Linear Pricing - average price paid by consumer changes based on quantity bought

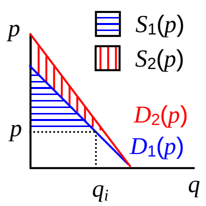
Assumptions -

1. Constant marginal cost (c)
2. Utility of consumer i characterized by parameter θ_i

   $$U = \begin{cases} \theta_i V(q) - T & \text{if consumer pays } T \text{ and consumes } q \text{ units of the good} \\ 0 & \text{if consumer doesn't buy} \end{cases}$$

   $V(q)$ is a "typical" utility function (diminishing marginal utility); $V' > 0$, $V'' < 0$, $V(0) = 0$

   Marginal Utility - $\frac{\partial U}{\partial q} = \theta_i V'(q)$ ... gives us the inverse demand curve $p_i = \theta_i V'(q)$

   No Income Effects - U independent of consumer's income; it's essentially quasi-linear utility

   Linear Demand - this assumption isn't necessary, but it's nice to have pictures to go along with all the math we're about to do

   $$V(q) = \frac{1-(1-q)^2}{2} \;\Rightarrow\; V'(q) = 1 - q$$

   Two Consumers - for our analysis we'll only be looking at two consumer types with $\theta_1 < \theta_2$ (i.e., type 2 values the good more

   Normalize Population - for simplicity, normalize population of consumers to 1 and define $\lambda \equiv$ proportion of θ_1 types $\Rightarrow 1 - \lambda \equiv$ proportion of θ_2 types

   Consumer Surplus - use same notation from previous section:

   $$S_i(p) = \int_p^\infty D_i(x)dx \;\Rightarrow\; S_i'(p) = -D_i(p) < 0 \text{ and } S_i''(p) = -D_i'(p) > 0$$

3. Monopolist knows distribution of demand, but can't distinguish whether a customer is of type 1 or type 2
4. Monopolist serves both types of consumers; this might seem artificially restrictive, but if he doesn't serve both we just have a single price monopoly; the two type example can be expanded to many types with a subset being served; the general result will still hold; one way to guarantee both markets are served is the have λ relatively large (kind of the Wal-Mart thing... low value customers, but lots of them)
5. Monopolist's profit function is well behaved (i.e. single peaked, concave function)

Two-Part Tariffs - this is a specific type of non-linear pricing; it's an artificial restriction on seller which we'll show soon isn't optimal, but it's easy to work with

$T(q) = A + pq$ ... A is fixed fee; p is price per unit

Consumer Problem - max $\theta_i V(q) - pq - A$

FOC - $\theta_i V'(q) - p = 0 \;\Rightarrow$ if A is set so that consumer participates (i.e., buys), then consumer's level of consumption is determined by his demand for the good at price p (i.e., MB = MC); this means there are no income effects from A

Firm Problem - $\max_{p,A} \pi^{SD} = A + \lambda(p-c)D_1(p) + (1-\lambda)(p-c)D_2(p)$

Individual Rationality - constraints for optimization: $S_i(p) - A \geq 0 \;\; \forall\, i$ (i.e., consumer will not buy if he ends up with a negative surplus); based on the way we set up this problem (i.e., serve all markets), the binding constraint will be the smallest consumer surplus so $A = \min[S_i(p)]$

Result - low value (low demand) customer gets zero consumer surplus

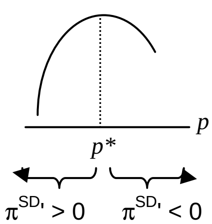
Optimal Price - $p^* =$ discriminating monopolist's price; $\bar{p} =$ nondiscriminating price

$\bar{p} > p^* > c$

<u>Proof</u>: $\pi^{SD}(p) = A + \lambda(p-c)D_1(p) + (1-\lambda)(p-c)D_2(p)$

Sub $A = S_1(p)$ and $\bar{\pi}(p) = \lambda(p-c)D_1(p) + (1-\lambda)(p-c)D_2(p)$ (single-price monopolist's profit):

$\pi^{SD}(p) = S_1(p) + \bar{\pi}(p)$

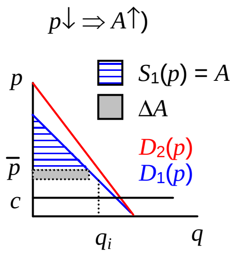


Take derivative wrt p and evaluate it at $\bar{p}$: $\pi^{SD}{}'(\bar{p}) = S_1{}'(\bar{p}) + \bar{\pi}'(\bar{p})$

Note that $\bar{\pi}'(\bar{p}) = 0$ (because $\bar{p}$ maximizes $\bar{\pi}(p)$)

Sub $S_1{}'(p) = -D_1(p) < 0$: $\pi^{SD}{}'(\bar{p}) = -D_1(\bar{p}) < 0$

$\therefore$ the slope of $\pi^{SD}(p)$ is negative at $\bar{p}$, so $\bar{p}$ is to the right of the maximum of $\pi^{SD}(p)$, which occurs at p^* so we showed $\bar{p} > p^*$



Now go back to $\pi^{SD}(p) = S_1(p) + \lambda(p-c)D_1(p) + (1-\lambda)(p-c)D_2(p)$

Factor out $(p-c)$: $\pi^{SD}(p) = S_1(p) + (p-c)[\lambda D_1(p) + (1-\lambda)D_2(p)]$

Take derivative wrt p:

$S_1{}'(\bar{p}) + [\lambda D_1(p) + (1-\lambda)D_2(p)] + [(p-c)\lambda D_1{}'(p) + (p-c)(1-\lambda)D_2{}'(p)]$

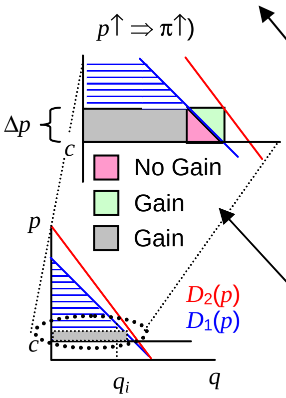
Evaluate at c:

$\pi^{SD}{}'(c) = S_1{}'(c) + [\lambda D_1(c) + (1-\lambda)D_2(c)] + [(c-c)\lambda D_1{}'(p) + (c-c)(1-\lambda)D_2{}'(c)]$

Sub $S_1{}'(c) = -D_1(c)$:

$\pi^{SD}{}'(c) = -D_1(c) + [\lambda D_1(c) + (1-\lambda)D_2(c)] = (1-\lambda)[D_2(c) - D_1(c)] > 0$

$\therefore$ the slope of $\pi^{SD}(p)$ is positive at c, so c is to the left of the maximum of $\pi^{SD}(p)$, which occurs at p^* so we showed $p^* > c$



Intuitive - (1) first order effect of Δp on $\bar{\pi}(p)$ is zero (i.e., $\bar{\pi}'(\bar{p}) = 0$) $\therefore p\!\downarrow$ on margin has no change in profit; but $p\!\downarrow$ increases $A = S_1(p)$ so firm is better off $\therefore \bar{p} > p^*$;

(2) if price is at c firm's profit is A; by raising price slightly, A will decrease by the gray and pink areas (the pink area is actually a "second-order small" area so we technically could ignore it); the higher price will return the gray area from type 1 consumers and the gray, pink and green areas from the type 2 consumers; if you add together the losses and gains, the firm sends up increasing profits by the sum of the gray and green areas

Welfare - relative to single-price monopoly, a monopoly using a two-part tariff that serves both types of customers will result in greater social welfare because it'll charge a lower price (which means a higher output) that brings it closer to the socially efficient outcome

Tie-In Sales - occurs with the basic good is consumed in a fixed quantity (usually just 1 unit), but there's a complementary good that's more closely linked to what is really being demanded... the services of the basic good (e.g., copier and copy paper... demand is for copies; cameras and film... demand is for pictures; printers and print cartridges... demand is for printed documents); if firm has a monopoly on both goods it can price discriminate using a two-part tariff where A is the cost of the basic good and p is the price of the complement; just like we showed above A will be equal to the consumer surplus of the lowest demand consumer the firm decides to serve; the cost of the basic good is irrelevant (like fixed costs) other than when the firm decides what level of consumer it'll serve (need cost > $S_i(p)$)

Competition Problem - if firm has monopoly on basic good, but not on the complement it's no longer able to price discriminate; on the plus side, a competitive market for the complement will result in p = c (i.e., consumer surplus increases) so firm can charge more for the basic good; of course, firm would do even better if it could price discriminate (i.e., had monopoly on both goods); the price of the basic good would fall, but the price of the complement will rise... one way firm's do this is by trying to set up the sale as a bundle (e.g., require Xerox paper only for the copier and set up a sales contract for both the copier and the paper)

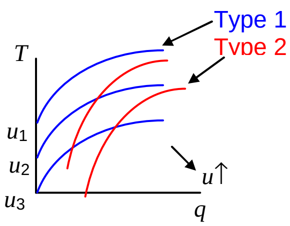
Legal Problem - people can always cheat on the agreement; anti-trust issues (technically the tie-in is intended to hurt competition which is one of the targets of anti-trust legislation)

Fully Non-Linear Pricing -

Graphic Representation - represent consumer utility, pricing policy, and firm profits per customer in (q,T) space

Consumer Utility -

Indifference Curves - Iso-utility curve; $U(q)=\theta_i V(q)-T=u_1$ (constant)



Look at $q\uparrow$ - increase quantity for given type of consumer

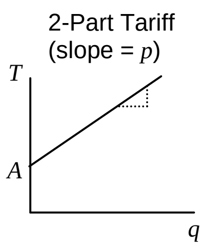
Totally differentiate: $\theta_i V'(q)dq-dT=0$

Solve for marginal benefit: $\dfrac{dT}{dq}=\theta_i V'(q)>0$ ... marginal willingness to pay

Take derivative wrt q: $\dfrac{d^2T}{dq^2}=\theta_i V''(q)<0$ ... diminishing returns (concave)

Look at $\theta\uparrow$ - look at different types of consumer

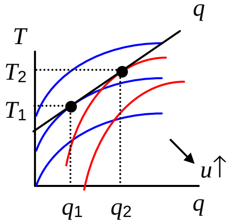
$\dfrac{d^2T}{dqd\theta}=V'(q)>0$ ... $\theta\uparrow \Rightarrow$ MB$\uparrow$... steeper marginal willingness to pay



Single Crossing - assume preferences have this property; any pair of indifference curves for two individuals will cross at most once; also called "sorting condition" or the "Spence-Mirrlees condition"

Link to Demand - single crossing in (q,T) space $\Leftrightarrow$ no crossing in (q, p) space (i.e., demand curves don't intersect)

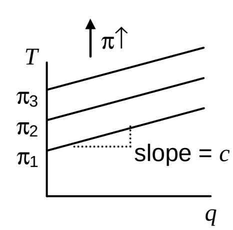
Pricing Policy - graph tariff (price) function; simple version is two-part tariff: $T(q)=A+pq$, but could be anything



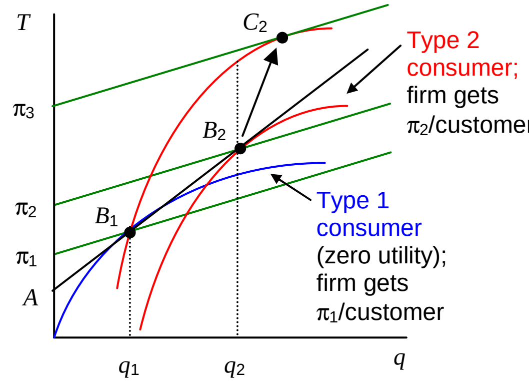
Consumer Optimum - consume at point where indifference curve is tangent to the price function

Note 1: $\theta\uparrow \Rightarrow q\uparrow$

Note 2: pricing scheme doesn't matter; could offer two bundles (T_1, q_1) and (T_2, q_2) and get same result (in general, one bundle per consumer)



Firm Profits - iso-profit per consumer: $\boldsymbol{\pi}(q)=T-cq=\boldsymbol{\pi}_1$ (constant)



Two-Part Tariff Not Optimal - look at (q,T) space representation of two-part tariff; type one consumers consume B_1 and type two consumers at B_2 giving firm π_1 and π_2 per customer (respectively); type 1 consumers have zero surplus so the firm can't squeeze anything out of them, but the firm could easily raise price to type 2 consumers to capture their utility; the ideal (i.e., max profit given B_1) would be to offer them C_2 which makes them indifferent to B_1 (or set price some small epsilon less than that so they strictly prefer C_2 over B_1); ∴ firm can make more profit than what they do with a two-part tariff

Mathematically - firm's problem is:

$$\max_{T_1,T_2,q_1,q_2} \boldsymbol{\pi} = \lambda(T_1 - cq_1) + (1-\lambda)(T_2 - cq_2)$$

s.t. $\theta_1 V(q_1) - T_1 \geq 0$ (IR1)

$\theta_2 V(q_2) - T_2 \geq 0$ (IR2)

$\theta_1 V(q_1) - T_1 \geq \theta_1 V(q_2) - T_2$ (IC1)

$\theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1$ (IC2)

Constraints -

Individual Rationality - consumer's utility is at least as big as the consumer's cost

Incentive Compatibility - consumer's net benefit (utility minus cost) for his designated bundle must be at least as great as his benefit for the other bundle

Facts -

1. IR1 binds and IR2 doesn't - type 1 (low demander) gets no surplus and type 2 gets some surplus

   Proof: Assume neither IR1 or IR2 binds, that means we can increase T_1 and T_2 by the same amount (so IC1 and IC2 are not affected) and do better... contradiction

   Assume IR1 doesn't bind; since at least one of them must bind IR2 must bind

   We know $\theta_2 V(q_1) > \theta_1 V(q_1)$, so from IR1 we can say $\theta_2 V(q_1) - T_1 \geq 0$

   Mix that with IC2 and we see that $\theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1 \geq 0$ which contradicts that IR2 is binding

   Assume IR1 and IR2 bind; that implies $\theta_1 V(q_1) - T_1 = 0$ and $\theta_2 V(q_2) - T_2 = 0$

   We know $\theta_2 V(q_1) > \theta_1 V(q_1)$, so from IR1 we can say $\theta_2 V(q_1) - T_1 \geq 0$

   Mix that with IC2 and we see that $\theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1 \geq 0$ which contradicts that IR2 is binding

   ∴ last option (IR1 binds and IR2 doesn't bind) must be true

2. IC2 binds and IC1 doesn't - seller doesn't have to worry about type 1 consumer going for the type 2 bundle, but does have to worry about type 2 consumers selecting the type 1 bundle

   Proof: Since IR2 doesn't bind, we can ignore that constraint (another one will be binding before IR2 becomes an issue)

   If IC2 doesn't bind we can increase T_2 (which relaxes IC1 and increases profit) indefinitely; ∴ IC2 must bind

   $\theta_2 V(q_2) - T_2 = \theta_2 V(q_1) - T_1 \Rightarrow \theta_2 (V(q_2) - V(q_1)) = T_2 - T_1$

   We know $\theta_2 V(q_1) > \theta_1 V(q_1)$, so $T_2 - T_1 > \theta_1 (V(q_2) - V(q_1))$

That can be rewritten $\theta_1 V(q_1) - T_1 > \theta_1(V(q_2) - T_2$ ... so IC1 doesn't bind

3. <u>Socially Optimal Output</u> - type 2 (high value) consumers purchase socially optimal quantity, but type 1 consumers purchase less than optimal quantity

Update Problem -

$$\max_{T_1,T_2,q_1,q_2} \boldsymbol{\pi} = \lambda(T_1 - cq_1) + (1-\lambda)(T_2 - cq_2)$$

$$\text{s.t. } \theta_1 V(q_1) - T_1 = 0 \qquad \text{(IR1)}$$

$$\theta_2 V(q_2) - T_2 = \theta_2 V(q_1) - T_1 \qquad \text{(IC2)}$$

Solve IR1 for T_1: $T_1 = \theta_1 V(q_1)$

Substitute T_1 into IC2: $\theta_2 V(q_2) - T_2 = \theta_2 V(q_1) - \theta_1 V(q_1)$

Solve IC2 for T_2: $T_2 = \theta_2(V(q_2) - V(q_1)) + \theta_1 V(q_1)$

Sub T_1 and T_2 into objective:

$$\max_{q_1,q_2} \boldsymbol{\pi} = \lambda(\theta_1 V(q_1) - cq_1) + (1-\lambda)(\theta_2(V(q_2) - V(q_1)) + \theta_1 V(q_1) - cq_2)$$

Now we have unconstrained optimization so we only need to look at FOC:

$$\frac{\partial \boldsymbol{\pi}}{\partial q_2} = (1-\lambda)(\theta_2 V'(q_2) - c) = 0 \;\Rightarrow\; 1-\lambda > 0 \;\therefore\; \boxed{\theta_2 V'(q_2) = c}$$

Economic Interpretation - MB = MC for type 2 consumer (firm doesn't distort type 2 consumer because he consumes socially optimal level)

$$\frac{\partial \boldsymbol{\pi}}{\partial q_1} = \lambda(\theta_1 V'(q_1) - c) + (1-\lambda)(\theta_1 - \theta_2)V'(q_1) = 0$$

Rearrange terms: $\theta_1 V'(q_1) - c = \dfrac{1-\lambda}{\lambda}(\theta_2 - \theta_1)V'(q_1)$

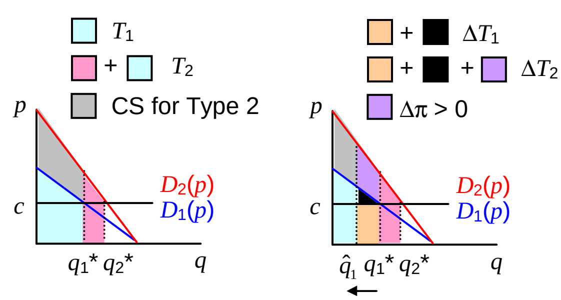
We know $\dfrac{1-\lambda}{\lambda} > 0$, $\theta_2 - \theta_1 > 0$, and $V'(q_1) > 0 \;\therefore\; \boxed{\theta_1 V'(q_1) - c > 0}$

Economic Interpretation - MB > MC for type 1 consumer; firm distorts type 1 consumer by giving him less than the socially optimal level

Intuition - look at pictures to explain the facts

Type 2 socially efficient (MB = MC)... if this wasn't the case, the firm would be "leaving money on the table" and could increase profit by capturing more of type 2's surplus; see the figure on the top of the previous page that shows C_2 as the optimal bundle for type 2 (given B_1 for type 1)

Type 1 distorted down (i.e., $\hat{q}_1 < q_1^*$ ... quantity with price discrimination < socially optimal quantity)



Left Graph - if type 1 gets q_1^*, his price (T_1) is the blue area; if type 2 selects type 1's bundle, his surplus is the gray area; ∴ firm is limited to charging type 2's; it must let them have the gray area under T_2 or they won't select that bundle (i.e. set T_2 to the pink and blue area)

Right Graph - suppose firm lowers the quantity to type 1 from q_1^* to $\hat{q}_1$; firm lowers T1 and loses the orange and black areas from type 1 (the orange is just cost saved, but the black is lost profit); but doing this the firm also reduced type 2's surplus from taking bundle 1; that frees the firm to raise T2 to include the orange, black, and purple areas; the first two areas make up for what was lost and the purple area is the net gain to the firm for lowering the quantity to type 1 consumers

Note: this intuitive explanation makes it seem like the firm can just ignore type 1 consumers and only serve type 2's, but that's misleading because we really didn't address the population density (λ) which means that black area may not be the same for type 1's and type 2's

Quality Discrimination

Basics - we can set this up so that it's exactly the same model (problem) as price discrimination; analogous conclusions

Model -

1. Let q denote the quality of the product
2. Constant marginal cost of quality (i.e., seller has cost cq for each unit produced, so marginal cost of quality is c)
3. Consumers have unit demands for good (utility doesn't rise if more than one unit is consumed); value of quality to consumer i characterized by parameter θ_i

$$U = \begin{cases} \theta_i V(q) - T & \text{if 1 unit is consumed} \\ 0 & \text{if consumer doesn't buy} \end{cases}$$

$V(q)$ is a "typical" utility function (diminishing marginal utility); $V' > 0$, $V'' < 0$, $V(0) = 0$

Two Consumers - for our analysis we'll only be looking at two consumer types with $\theta_1 < \theta_2$ (i.e., type 2 values the good more

Results - "widening of the quality spectrum"

1. First best optimal quality to type 2 consumer
2. Quality for type 1 consumer is distorted down (less than socially optimal)

Example - railroads, transatlantic ships, airlines... first vs. second class; make second class lousy; firms charge less to second class customers, but make up for it by having less first class customers go to second class

General - all the price discrimination topics we covered generally hold for many types of customers, not just 2

Related Problems - any type of **adverse selection**

Insurance - less healthy (high demand) vs. healthy (low demand) customers; to keep less healthy from getting healthy person's policy, company drops quality of healthy policy (less coverage)

Short-Run Price Competition
(Tirole Chpt 5)

Topics to cover...
- Intro... done
- Cournot Model (quantity)... done
- Bertrand Model (prices)
- Two-Stage (capacity then prices) - rationalization of Cournot

Short-Run - firms interact just once; no dynamics (until we get to two-stage)

Cournot Model

Assumptions -

2 Firms with homogeneous product

Firms choose <u>quantities</u> simultaneously & assume respective market clearing price will result (doesn't really discuss how the price is set)

Equilibrium exists... we'll cover existence criteria later

Firm *i*'s profit - $\pi^i(q_1, q_2) = P(q_1 + q_2) \cdot q_i - C_i(q_i)$

Weakly Concave Demand - $P' < 0$ and $P'' \leq 0$

Weakly Convex Cost - $C'' \geq 0$

Nash Equilibrium - output pair that solves both reaction functions; one graph, point where reaction functions intersect (Note: we're only looking at pure strategy equilibria)

Reaction Function - best reply function; returns q_i that maximizes π^i given q_j

$$R_i(q_j) \equiv q_i^* \text{ s.t. } \pi_i^i = \frac{\partial \pi^i}{\partial q_i} = P'(q_i + q_j) \cdot q_i + P(q_i + q_j) - C_i'(q_i) = 0$$

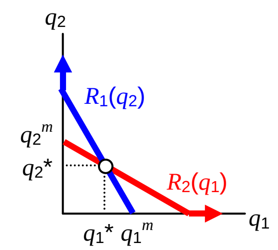
Graph - to plot $R_1(q_2)$, we need to know the slop so totally differentiate

$\pi_1^1(R_1, q_2) = 0$:

$$\pi_{11}^1 dR_1 + \pi_{12}^1 dq_2) = 0 \Rightarrow \frac{dR_1}{dq_2} = -\frac{\pi_{12}^1}{\pi_{11}^1} = -\frac{P''q_1 + P'}{P''q_1 + 2P' - C_1''} < 0$$

Slope is < 0 because $P' < 0$, $P'' \leq 0$, $C'' \geq 0$ (i.e., both numerator and denominator are < 0 and the ratio is multiplied by a negative)

For $R_2(q_1)$, just swap the 1's and 2's above

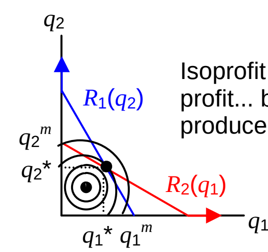


Monopoly output for firm 1 (because $q_2 = 0$)

Externality - there's an externality between (across) the firms in the case where both firms produce (i.e., costs aren't too different)... price and joint profits are lower than monopoly (cartel) price and profits

<u>Proof</u>: Look at firm *i*: $\frac{\partial \pi^i}{\partial q_i} = P' \cdot q_i + P - C_i'(q_i) = 0$

Look at cartel: $\frac{\partial \pi^i}{\partial q_i} + \frac{\partial \pi^j}{\partial q_i} = P' \cdot (q_i + q_j) + P - C_i'(q_i) = 0$



Isoprofit curves for joint profit... both firms produce too much

Note the extra term in the cartel FOC: $q_j > 0$ so if we evaluate at q_i* we get

$\frac{\partial \pi^i}{\partial q_i} + \frac{\partial \pi^j}{\partial q_i} = P' \cdot q_j < 0$... which means joint profits are decreasing in q_i (i.e., joint profits will rise if firm i decreases quantity below q_i*)

Externality occurs because firm i only looks at $\partial \pi^i / \partial q_i$ and doesn't consider how its quantity decision affects π_j

Production Cost - if firms have different cost functions, then they (almost always) will not minimize total cost of production

Proof: joint cost minimization is min $C_i + C_j$ s.t. $q_i + q_j =$ constant $\Rightarrow$ $C_i' = C_j'$ (assuming interior solution... both firms produce)

Cournot FOC (rearranged from prev. page): $P - C_i' = -P' \cdot q_i$ and $P - C_j' = -P' \cdot q_j$...

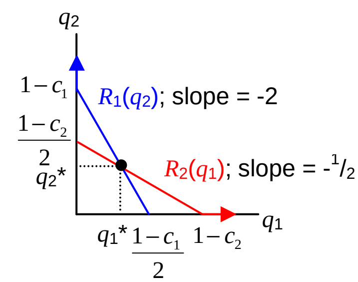
since both conditions have the same P and P', the only way we can have $C_i' = C_j'$ is if $q_i = q_j$... the only way occurs in Cournot equilibrium is if the firms have the same cost functions

Social Welfare - Cournot equilibrium is socially inefficient

Not First Best - look at FOC: $P - C_i' = -P' \cdot q_i > 0$ $\Rightarrow$ $P \neq C_i'$... price (marginal benefit) is not equal to marginal cost; price is too high relative to socially efficient (competitive) price

Not Second Best - given output under Cournot equilibrium, there's a cost inefficiency

Linear Case - linear demand and total cost (i.e., constant marginal cost)



$P(q) = 1 - q_1 - q_2$ and $C_i(q_i) = c_i q_i$ ($c_i \neq c_j$ is allowed)

Find R_i - $\max_{q_i} \pi^i = (1 - q_i - q_j) q_i - c_i q_i$

FOC: $\frac{\partial \pi^i}{\partial q_i} = 1 - q_j - 2q_i - c_i = 0 \Rightarrow q_i* = \boxed{R_i(q_j) = \frac{1 - q_j - c_i}{2}}$

Equilibrium - solve $q_1* = \frac{1 - q*_2 - c_1}{2}$ and $q_2* = \frac{1 - q*_1 - c_2}{2}$

$$q_1* = \frac{1 - \frac{1 - q_1* - c_2}{2} - c_1}{2} = \frac{1}{2} - \frac{1}{4} + \frac{q_1*}{4} + \frac{c_2}{4} - \frac{c_1}{2} \Rightarrow \frac{3}{4} q_1* = \frac{1 - 2c_1 + c_2}{4} \Rightarrow$$

$\boxed{q_1* = \frac{1 - 2c_1 + c_2}{3}}$... swap 1 and 2 for q_2*

$$\pi^1 = \left(1 - \frac{1 - 2c_1 + c_2}{3} - \frac{1 - 2c_2 + c_1}{3}\right) \frac{1 - 2c_1 + c_2}{3} - c_1 \frac{1 - 2c_1 + c_2}{3} =$$

$$\left(\frac{3 - 1 + 2c_1 - c_2 - 1 + 2c_2 - c_1}{3}\right) \frac{1 - 2c_1 + c_2}{3} - \frac{c_1 - 2c_1^2 + c_1 c_2}{3} =$$

$$\left(\frac{1 + c_1 + c_2}{3}\right) \frac{1 - 2c_1 + c_2}{3} + \frac{-3c_1 + 6c_1^2 - 3c_1 c_2}{9} =$$

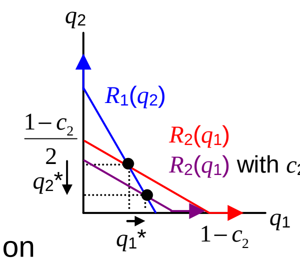
$$\frac{1 - 2c_1 + c_2 + c_1 - 2c_1^2 + c_1 c_2 + c_2 - 2c_1 c_2 + c_2^2}{9} + \frac{-3c_1 + 6c_1^2 - 3c_1 c_2}{9} =$$

$$\frac{(1-2c_1+c_2)+(-2c_1+4c_1^2-2c_1c_2)+(c_2-2c_1c_2+c_2^2)}{9}=$$

$$\frac{(1-2c_1+c_2)-2c_1(1-2c_1+c_2)+c_2(1-2c_1+c_2)}{9}=\boxed{\frac{(1-2c_1+c_2)^2}{9}}$$ ... enough algebra for you?... swap 1 and 2 for π^2

Will always have existence (lines will cross) and uniqueness (only cross once), although it could be at an axis so one firm doesn't produce; we'll address these in more detail later

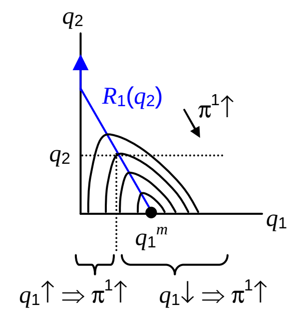
Comparative Statics -



$$\frac{\partial q_1{*}}{\partial c_2}=\frac{1}{3}>0 \;\therefore\; \text{firm 2's cost} \uparrow \Rightarrow \text{firm 1's quantity} \uparrow$$

$$\frac{\partial \pi^1}{\partial c_2}=\frac{2(1-2c_1+c_2)}{9}>0 \;\therefore\; \text{firm 2's cost} \uparrow \Rightarrow \text{firm 1's profit} \uparrow$$

Isoprofit for Firm 1 - $\pi^1(q_1,q_2)=$ constant (if P < MC); slope equals zero on $R_1(q_2)$ by definition



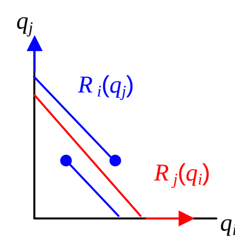
Note: at given level of q_2, if $q_1 < q_1{*}$, then increasing q_1 will increase profits to firm 1; similarly, if $q_1 > q_1{*}$, then decreasing q_1 will increase profits (in both cases, firm 1 would be moving towards it's best reply function, $R_1(q_2)$)

Existence - idea (not proof) for standard approach with 2 firms

Assume $\pi^i(q_i,q_j)$ is continuous in (q_i,q_j) (implied by $P(q_i+q_j)$ and $C_i(q_i)$ being continuous)

Assume monopoly outputs are positive (i.e., $P(0)>C_i(0)$)

If $\pi^i(q_i,q_j)$ is concave in q_i (for any q_j) then equilibrium exists because $R_i(q_j)$ is continuous and eventually decreases... application of fixed-point theorem (the two best reply functions will cross)

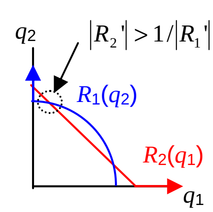


Suppose $\pi^i(q_i,q_j)$ is not concave in q_i; this allows the function to be double peaked which could result in $R_i(q_j)$ not being continuous and allow $R_j(q_i)$ to not cross

Uniqueness - Assume existence; equilibrium may not be unique

Sufficient Condition - assume there are 2 firms with multiple equilibria; in at least one point, R_2 will be flatter than R_1... i.e., $|R_2'|>1/|R_1'| \;\Rightarrow\; |R_1'||R_2'|>1 \;\Rightarrow\; |R_i'(q_j)|<1$

Sufficient for Sufficient - $R_i'=\dfrac{dR_i}{dq_j}=-\dfrac{\pi^i_{ij}}{\pi^i_{ii}}$ (found this on p.1)



$$R_i'=-\frac{P''q_i+P'}{P''q_i+2P'-C_i''}=\frac{P''q_i+P'}{-\underbrace{(P''q_i+P')}_{<0}+\underbrace{(P'-C_i'')}_{<0}}$$

$P''\le 0$ and $C_i''\ge 0$ is a sufficient condition for $|R_i'|<1$ $\therefore$ assumptions of Cournot model guarantee unique equilibrium exists

Cournot with *n* Firms

Assume existence and uniqueness of equilibrium
Assume symmetric firms (same cost)
Linear Case -

Inverse Demand - $P(Q) = 1 - Q = 1 - \sum_{i=1}^{n} q_i$

Cost - $C_i(q_i) = cq_i$ ... constant marginal cost, c, same for all firms... assume $P(0) > c$

Profit for Firm *i* - $\pi^i = \left(1 - \sum_{j=1}^{n} q_j\right) q_i - cq_i$

FOC - $\frac{\partial \pi^i}{\partial q_i} = 1 - \sum_{j \neq i} q_j - 2q_i - c = 0$

Symmetry - since firms have same cost, equilibrium will be symmetric (all q_i the same... let it be the Cournot Output: q_c)... we'll prove this on HW... $\therefore$

$\frac{\partial \pi^i}{\partial q_i} = 1 - (n-1)q_c - 2q_c - c = 0 \Rightarrow \boxed{q_c = \frac{1-c}{n+1}}$ ... note: as $n \to \infty$, $q_c \to 0$

Cournot Price - $p_c = 1 - \sum_{i=1}^{n} q_c = 1 - n\frac{1-c}{n+1} = \frac{n+1}{n+1} - \frac{n}{n+1} + \frac{nc}{n+1}$

Combine first two terms and multiply first term by $\frac{1}{n} / \frac{1}{n}$: $\boxed{p_c = \frac{1}{n+1} + \frac{c}{1+1/n}}$

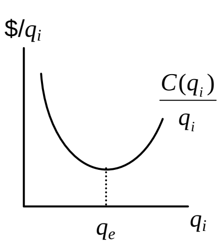
$\lim_{n \to \infty} p_c = \lim_{n \to \infty}\left(\frac{1}{n+1} + \frac{c}{1+1/n}\right) = c$ ... competitive outcome

Intuition - general FOC for Cournot firm: $\frac{\partial \pi^i}{\partial q_i} = P' \cdot q_i + P - C_i'(q_i) = 0$ ... market power is captured by $P' \cdot q_i$ term; if there's fixed market demand, $n\uparrow \Rightarrow q_i\downarrow \Rightarrow P' \cdot q_i \downarrow$ so firm's "lose" market power and market becomes more like competitive market

Free Entry Cournot Equilibrium - previous analysis assumed n was exogenous (didn't say why n was getting bigger, just assumed it); now we'll use free entry condition to make n endogenous and explain why n gets bigger or smaller... still get same result: larger n results in p closer to c (competitive or efficient equilibrium)

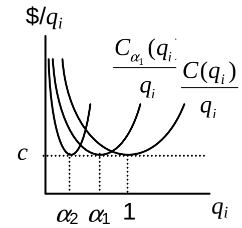
1 Stage Game - firms simultaneous decide whether to enter and how much to produce (HW will look at entry decision separate from production decision)

Assumptions -



1. As many equally efficient producers as want to produce
2. n is number of active (producing) firms
3. Every firm has same cost function, $C(q_i)$
4. Cournot equilibrium with n producing firms must have $\pi^i \geq 0$ for all n firms
5. No firm wants to enter taking market output as given (Cournot assumption) because no new firm would be profitable
6. Average total cost is "U-shaped"... lots of empirical evidence to support this (initially have increases returns, then have decreasing returns)

    Efficient Scale - output that minimizes average total cost



7. Demand given by $P(Q)$
8. Equilibrium exists
9. Q^* is "competitive output" (i.e., $P(Q^*) = c$)

Issues -
(a) what happens to p as n gets large?
(b) how does n get large?

Normalize Efficient Scale - $\min_q \frac{C(q)}{q} = c$ and $\arg\min_q \frac{C(q)}{q} = 1$

Parameterize Cost - use $\alpha \in (0,1]$ such that $C_\alpha(q) \equiv \alpha C(q/\alpha)$

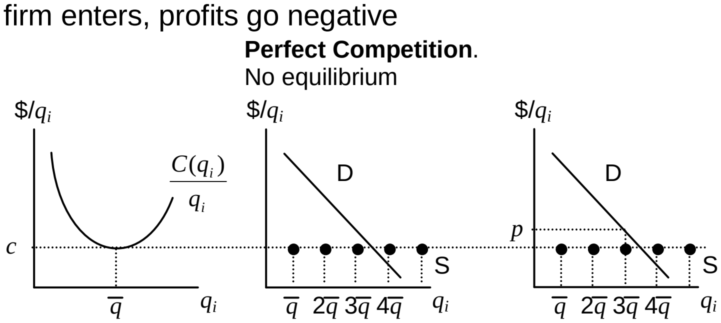
$$\min_q \frac{C_\alpha(q)}{q} = \min_q \frac{\alpha C(q/\alpha)}{q} = \min_q \frac{C(q/\alpha)}{q/\alpha} = c$$

$\arg\min_q \frac{C(q)}{q} = \arg\min_q \frac{C(q/\alpha)}{q/\alpha} = \alpha$ ... moves ATC left with minimum value still c

Result - equilibrium output is in interval $[Q^* - \alpha, Q^*]$
Proof: $Q > Q^* \Rightarrow P(Q) < c \Rightarrow \pi < 0$ (can't be an equilibrium)
$Q < Q^* - \alpha \Rightarrow$ firm could enter, produce $q_i = \alpha$ and be profitable
Firm's ATC if it produces α is c; price $P(Q + \alpha) > P(Q^*) = c$ since $Q + \alpha < Q^*$
(comes directly from $Q < Q^* - \alpha$) $\therefore \pi^i > 0$ so this isn't an equilibrium
$\therefore Q \in [Q^* - \alpha, Q^*]$

Corollary - (a) as $\alpha \to 0$, $Q \to Q^*$ and (b) as $\alpha \to 0$, $P \to c$

Link to Perfect Competition - this solves the existence problem of the perfectly competitive model; in that case, perfect competition has conflicting requirements for equilibrium: zero profit, supply = demand, and finite number of firms; in general supply is discrete because n is discrete; Cournot Solution says number of firms produce with positive profit, but if one more firm enters, profits go negative



Cournot Competition.
Equilibrium: $n = 3$; $p > c$ so firms make a small positive profit;
If $n = 4$, $p < c$ so firms make negative profit... not an equilibrium
Note: $\alpha\downarrow \Rightarrow$ dots are getting close together so we get equilibrium closer to $p = c$

Bertrand Oligopoly

Assumptions -

2 Firms with homogeneous product
Firms choose <u>prices</u> simultaneously, then satisfy forthcoming demand as they please
Constant marginal cost c (same for both firms)
No fixed cost (or they're already spent [sunk] $\therefore$ irrelevant)

Result - $p = c$ and $\pi^i = 0$... same as competitive result with just 2 firms (also holds for $n > 2$)

<u>Proof</u>: need notation first

$D(p)$ is market demand; $D_i(p_i, p_j)$ is demand faced by firm i

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j \\ \tfrac{1}{2}D(p_i) & \text{if } p_i = p_j \text{ (assumption; exactly spit not important)} \\ 0 & \text{if } p_i > p_j \end{cases}$$

Need to show unique equilibrium has $p_i^* = p_j^* = c$

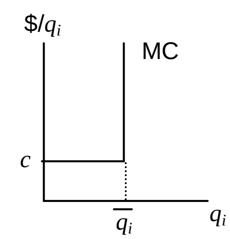
Ignore all cases with $p < c$ (firms won't produce with negative profit)

a. $p_i^* > p_j^* > c$ ... $\pi^i = 0$, but firm i could drop price between p_j^* and c and make positive profit $\therefore$ p_i^* is not best reply to p_j^* (i.e., not equilibrium)

b. $p_i^* > p_j^* = c$ ... $\pi^j = 0$, but firm j could raise price between p_i^* and c and make positive profit $\therefore$ p_j^* is not best reply to p_i^* (i.e., not equilibrium)

c. $p_i^* = p_j^* > c$ ... $\pi^i = \pi^j > 0$, but firm either firm could drop price small amount and capture all demand: $D(p_i^* - \varepsilon)(p_i^* - \varepsilon - c) > \frac{1}{2}D(p_i^*)(p_i^* - c)$ $\therefore$ p_i^* is not best reply to p_j^* (i.e., not equilibrium)

d. $p_i^* = p_j^* = c$ ... equilibrium because no firm can do better given other firm's price (i.e., both firms on best reply function)



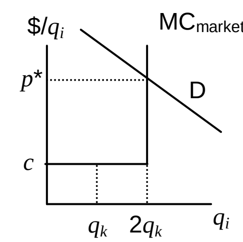
Solutions to Bertrand Paradox - paradox is that 2 firms will charge competitive price

Repeated Interaction - could get cooperation to cartel solution (monopoly price)

Product Differentiation - cutting price below competitor may not take all demand

Capacity Constraint - limits how much firm can sell; with constant marginal cost greater than market price, firm will want to sell everything it can

Decreasing Returns - capacity constraint is extreme case of decreasing returns (or increasing MC), but the general case isn't guaranteed to have an equilibrium like capacity constraint is

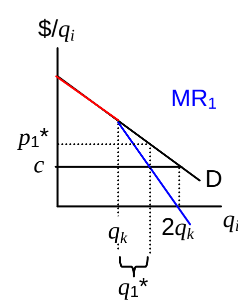


Equal Capacities - suppose firms have same capacity well below market demand... graph should be clear that price will be at least $p^* > c$

Residual Demand - exact solution depends on the character of the residual demand (demand faced by firm charging higher price after lower price firm serves up to its capacity)

Rationing Rules - 2 in the book: efficient and proportional; we'll only cover efficient

Edgeworth (1987) - assumed both firms had same capacity q_k with $D(c) = 2q_k$; suppose firm 1 raises price above c; everyone wants to buy from firm 2; assume high demanders (red part of demand curve) buy from two (efficient rationing); firm 1 now behaves like a monopoly with its residual demand (black portion) and produces q_1^* at $p_1^* > c$



Summary of Last Day's Notes

2 Stage Competition - firms choose capacities, then prices; solves problem of Cournot model which doesn't discuss prices, but gives Cournot outcome

Efficient Rationing - tells how good is rationed (offered to different consumers) when firms charge different prices and lower price firm doesn't want to satisfy all market demand

Consider firm 1 as low price firm ($p_1 < p_2$); it has capacity $\bar{q}_1$ and MC = 0 (could use MC = c, but result is the same and using zero is easier); also assume $p_1 < P(\bar{q}_1)$

Efficient Rationing - demander with highest value for product gets it form lower price firm

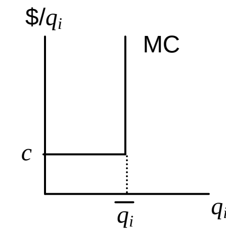
Residual Demand DR(p2) - demand faced by firm 2 after firm 1 sells its capacity

$$D_R(p_2) = \begin{cases} D(p_2) - \bar{q}_1 & \text{if } D(p_2) \geq \bar{q}_1 \\ 0 & \text{else} \end{cases}$$

Cournot - firm 2 essentially choosing q_2 knowing that price will be determined by $D_R(p_2)$; "in effect" this is equivalent to taking $\bar{q}_1$ as given and choosing q_2 expecting the market clearing price... that's the Cournot model

Two-Stage Game - Stage 1 - choose $\bar{q}_1$ and $\bar{q}_2$ simultaneously with MC = c_0

Stage 2 - set prices simultaneously (c_0 is sunk cost so MC = 0 up to capacity constraint)



$$c_i(q_i) = \begin{cases} 0 & \text{if } q_i \leq \bar{q}_i \\ \infty & \text{if } q_i > \bar{q}_i \end{cases}$$

Assume efficient rationing and look for subgame perfect pure strategy Nash equilibrium...

1) look at stage 2 first taking $\bar{q}_1$ and $\bar{q}_2$ as given

Result - if capacities are "not too different" and "not too big" (i.e., smaller relative to market demand), then equilibrium is $p_1 = p_2 = P(\bar{q}_1 + \bar{q}_2)$ with firms selling to capacity

"Not Too Big" - pure strategy equilibrium exists *iff* $\bar{q}_i \leq R_i(\bar{q}_i)$